Test code: ME I/ME II, 2004
Syllabus for ME I

Matrix Algebra: Matrices and Vectors, Matrix Operations, Determinants,

Nonsingularity, Inversion, Cramer's rule.

Calculus: Limits, Continuity, Differentiation of functions of one or more variables, Product rule, Partial and total derivatives,Derivatives of implicit functions, Unconstrained optimization (first and second order conditions for extrema of a single variable and several variables). Taylor Series, Definite and Indefinite Integrals: standard formulae, integration by parts and integration by substitution. Differential equations. Constrained optimization of functions of a single variable.

Theory of Sequence and Series:

Linear Programming: Formulations, statements of Primal and Dual problems. Graphical Solutions.

Theory of Polynomial Equations (up to third degree)

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory.

Sample Questions for ME I (Mathematics), 2004

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

1. $X \sim B(n, p)$. The maximum value of Var (X) is

   (A) $\frac{n}{4}$; (B) n; (C) $\frac{n}{2}$; (D) $\frac{1}{n}$.

2. P (x) is a quadratic polynomial such that P (1) = - P (2). If one root of the equation is –1, the other root is

   (A) $-\frac{4}{5}$; (B) $\frac{8}{5}$; (C) $\frac{4}{5}$; (D) $-\frac{8}{5}$.

3. $f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3$, $a < b < c$.

   The number of real roots of $f(x) = 0$ is

   (A) 3; (B) 2; (C) 1; (D) 0.

4. A problem of statistics is given to the three students A, B and C. Their probabilities of solving it independently are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, respectively. The probability that the problem will be solved is

   (A) $\frac{3}{5}$; (B) $\frac{2}{5}$; (C) $\frac{1}{5}$; (D) $\frac{4}{5}$.

5. Suppose correlation coefficients between x and y are computed from (i) $y = 2 + 3x$ and (ii) $2y = 5 + 8x$. Call them ρ_1 and ρ_2, respectively.

   Then

(A) $\rho_1 > \rho_2$; (B) $\rho_2 > \rho_1$; (C) $\rho_1 = \rho_2$; (D) either $\rho_1 > \rho_2$ or $\rho_1 < \rho_2$.

6. In the linear regression of y on x, the estimate of the slope parameter is given by $\frac{Cov(x,y)}{V(x)}$. Then the slope parameter for the linear regression of x on y is given by

(A) $\frac{V(x)}{Cov(x,y)}$; (B) $\frac{Cov(x,y)}{V(x)}$; (C) $\frac{Cov(x,y)}{\sqrt{V(x)V(y)}}$; (D) none of these.

7. Suppose $f(x) = e^x$ then

(A) $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(B) $f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(C) $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

$f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

(D) there exists at least one pair (x_1, x_2), $x_1 \neq x_2$ such that

$$f\left(\frac{x_1+x_2}{2}\right) = \frac{f(x_1)+f(x_2)}{2}$$

8. Consider the series (i) and (ii) defined below:

(i) $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\dots$

and

(ii) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$

Then,

(A) the first series converges, but the second series does not converge;

(B) the second series converges, but the first series does not converge;

(C) both converge;

(D) both diverge.

9. The function $x|x|$ is

(A) discontinuous at $x = 0$;

(B) continuous but not differentiable at $x = 0$;

(C) differentiable at $x = 0$;

(D) continuous everywhere but not differentiable anywhere.

10. The sequence $(-1)^{n+1}$ has

(A) no limit;

(B) 1 as the limit;

(C) –1 as the limit;

(D) 1 and –1 as the limits.

11. The series

$1.\frac{1}{2}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{4}+\ldots+\frac{1}{n}.\frac{1}{n+1}+\ldots$

(A) diverges;

(B) converges to a number between 0 and 1;

(C) converges to a number greater than 2;

(D) none of these.

12. Consider the function

$$f(x)=\frac{x^t-1}{x^t+1}, \qquad (x>0)$$

The limit of the function as t tends to infinity;

(A) does not exist; (B) exists and is everywhere continuous;

(C) exists and is discontinuous at exactly one point;

(D) exists and is discontinuous at exactly two points.

13. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}$ is equal to

(A) *sin u cos u* ; (B) *cot u* ; (C) *tan u* ; (D) none of these.

14. The deviative of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ with respect to x is

(A) $-\frac{x}{2}$; (B) $\frac{x}{2}$; (C) $\frac{1}{2}$; (D) $-\frac{1}{2}$.

15. The sum of the infinite series $1+\frac{3}{4}+\frac{7}{16}+\frac{15}{64}+\frac{31}{256}+\ldots$ is

(A) $\frac{2}{3}$; (B) $\frac{4}{3}$; (C) $\frac{8}{3}$; (D) none of these.

16. Five boys and four girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is

(A) $6_{P_4}\times\lfloor 5$; (B) $4_{P_2}\times\lfloor 5$; (C) $\lfloor 4\times\lfloor 5$; (D) none of these.

17. A point moves so that the ratio of its distance from the points *(-a, 0)* and *(a, 0)* is 2:3. The equation of its locus is

(A) $x^2+y^2+10ax+a^2=0$;

(B) $5x^2+5y^2+26ax+5a^2=0$;

(C) $5x^2+5y^2-26ax+5a^2=0$;

(D) $x^2 + y^2 - 10ax + a^2 = 0$;

18. If the sum, $\sum_{x=1}^{100} \lfloor x$, is divided by 36, the remainder is

(A) 3; (B) 6; (C) 9; (D) none of these.

19. If a, b, c, d are in G.P., then $(a^3 + b^3)^{-1}, (b^3 + c^3)^{-1}, (c^3 + d^3)^{-1}$ are in

(A) A. P.; (B) G. P.; (C) H.P.; (D) none of these.

20. The solution set of the inequality $||x| - 1| < 1 - x$ is

(A) $(-\infty, 0)$; (B) $(-\infty, \infty)$; (C) $(0, \infty)$; (D) $(-1, 1)$.

21. The distance of the curve, $y = x^2$, from the straight line $2x - y = 4$ is minimum at the point

(A) $(-1, 1)$; (B) $(1, 1)$; (C) $(2, 4)$; (D) $\left(\frac{1}{2}, \frac{1}{4}\right)$

22. The dual to the following linear program:

$$\begin{aligned} &\text{maximise} && x_1 + x_2 \\ &\text{subject to} && -3x_1 + 2x_2 \le -1 \\ &&& x_1 - x_2 \le 2 \\ &&& x_1 \ge 0, x_2 \ge 0 \end{aligned}$$

has

(A) a unique optimal solution;

(B) a feasible solution, but no optimal solution;

(C) multiple optimal solutions;

(D) no feasible solution.

23. The number of real roots of the equation

$x^2 - 3|x| + 2 = 0$

is

(A) 1; (B) 2; (C) 3; (D) 4.

24. There are four letters and four directed envelopes. The number of ways in which the letters can be put into the envelopes so that every letter is in a wrong envelope is

(A) 9; (B) 12; (C) 16; (D) 64.

25. If $a^2x^2 + 2bx + c = 0$ has one root greater than unity and the other less than unity, then

(A) $a^2 + 2b + c = 0$; (B) $a^2 + 2b + c > 0$;

(C) $2b + c < 0$; (D) $2b + c > 0$;

26. Given the two sequences $a_n = \frac{1}{n}$ and

$b_n = \frac{1}{n+1}$, the sum, $\sum_{n=1}^{99} \frac{(a_n - b_n)^2}{a_n b_n}$, is

(A) 1; (B) $1 - \frac{1}{99}$; (C) $\frac{99}{100}$; (D) none of these.

27. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^5$ is

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; (B) $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$; (C) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$; (D) none of these.

28. $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ is

(A) 0; (B) 1; (C) $e^{-\frac{1}{2}}$; (D) $e^{-\frac{1}{6}}$.

29. The system of equations

$$x + y + z = 6$$
$$x + ay - z = 1$$
$$2x + 2y + bz = 12$$

has a unique solution if and only if

(A) $a \neq 1$; (B) $b \neq 2$; (C) $ab \neq 2a + b + 2$; (D) none of these.

30. The number of times $y = x^3 - 3x + 3$ intersects the $x-$axis is

(A) 0; (B) 1; (C) 2; (D) 3.

Syllabus for ME II (Economics)

Microeconomics: Theory of consumer behaviour, Theory of producer behaviour, Market forms and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model, Aggregate demand and aggregate supply model, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2004

NO. 1. ***Instruction for question numbers 1 (i) – 1 (vi)***
For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in you answer book.

(i) A consumer consumes only two goods x and y. Her utility function is $U(x, y) = x + y$. Her budget constraint is $px + y = 10$ where p is the price of good x. If $p = \frac{1}{2}$, then the (absolute) own – price elasticity of good x is

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) ∞

(ii) A consumer consumes only two goods x and y. The price of good x in the local market is p and that in a distant market is q, where $p > q$. However, to go to the distant market, the consumer has to incur a fixed cost C. Suppose that the price of good y is unity in both markets. The consumer's income is I and $I > C$. Let x_0 be the equilibrium consumption of good x. If the consumer has smooth downward sloping and convex indiference curves, then

(A) $(p-q)x_0 = C$ always holds;

(B) $(p-q)x_0 = C$ never holds;

(C) $(p-q)x_0 = C$ may or may not hold depending on the consumer's preferences;

(D) none of the above.

(iii) Consider the following production function $Q = \min\left(\frac{L}{2a}, \frac{K}{4b}\right)$.

Let w and r be the wage and rental rate respectively. The cost function associated with this production function is

(A) 2 awQ;

(B) 4 brQ;

(C) $(wa + 2br)Q$;

(D) none of the above.

(iv) During a period net loan from abroad of an economy is positive. This necessarily implies that during this period

(A) trade balance is positive;

(B) net factor income from abroad is negative;

(C) current account surplus is negative;

(D) change in foreign exchange reserve is positive.

(v) Consider a simple Keynesian economy in which the government expenditure (G) exactly equals its total tax revenue: $G = tY$ where t is the tax rate and Y is the national income. Suppose that the government raises t. Then

(A) Y increases;

(B) Y decreases;

(C) Y remains unchanged;

(D) Y may increase or decrease.

(vi) Which one of the following statements is FALSE?

Interest on pubic debt is not a part of

(A) both personal income and national income;

(B) government consumption expenditure;

(C) national income;

(D) personal income.

No. 2 Indicate whether the following statements are TRUE or FALSE, adding a few lines to justify your answer in each case:

(i) A barrel of crude oil yields a fixed number of gallons of gasoline. Therefore, the price per gallon of gasoline divided by the price per barrel of crude oil is independent of crude oil production.

(ii) If there is no money illusion, once you know all the price elasticities of demand for a commodity, you can calculate its income elasticity.

(iii) If two agents for an Edgeworth box diagram have homothetic preferences then the contract curve is a straight line joining the two origins.

No. 3 Consider a duopoly situation where the inverse market demand function is P(Q) = 10- Q $(\text{where } Q = q_1 + q_2)$. The cost function of firm 1 is $(1+2q_1)$ and that of firm 2 is $(1+4q_2)$. The firms do not incur any fixed cost if they produce nothing. Calculate the Cournot equilibrium output and profit of the two firms. If, ceteris paribus, the fixed cost of firm 2 is Rs. 2 (instead of Re. 1), what happens to the Cournot equilibrium?

No. 4 A simple Keynesian model has two groups of income earners. The income of group 1 (Y_1) is fixed at Rs. 800. Both groups have proportional consumption function; the average propensity to consume is 0.8 for group 1 and 0.5 for group 2. Group 2 consumes only domestically produced goods. However, group 1 consumes both domestically produced as well as imported goods, their marginal propensity to import being 0.4. investment goods are produced domestically and Investment (I) is autonomously given at Rs. 600.

(i) Compute gross domestic product (Y)

(ii) Suppose group 2 makes an income transfer of Rs. 100 to group 1. However, imports are restricted and cannot exceed Rs. 250 (that is, import function ceases to be operative at this value). How does Y change?

(iii) How does your answer to part (ii) change, if the upper limit of imports is raised to Rs. 400?

No. 5 A consumer consumes electricity (X_E) and other goods (X_O). The price of other goods is unity. To consume electricity the consumer has to pay a rental charge R and a per unit price p. However, p increases with the quantity of electricity consumed according to the function $p = \frac{1}{2} X_E$. The utility function of the consumer is $U = X_E + X_O$ and his income is $I > R$.

(i) Draw the budget line of the consumer.

(ii) If $R = 0$ and $I = 1$, find the optimum consumption bundle.

(iii) Find the maximum R that the electricity company can extract from the consumer.

No. 6 A consumer has Rs. 25 to spend on two goods x and y. The price of good x is Rs. 3 and that of good y is Rs. 4. The continuously differentiable utility function of the consumer is $U(x, y) = 12x + 16y - x^2 - y^2$ where $x \geq 0$ and $y \geq 0$. What happens to the optimum commodity bundle if, instead of Rs. 25, the consumer has Rs. 50 or more to spend on the two goods?

No. 7 Consider an IS – LM model with the following elements:

$$s = s\left(y - t.\theta.y\right), \qquad 0 < s'(.) < 1 \qquad (1)$$

$$i = i\,(r), \quad i'(.) < 0 \qquad (2)$$

$$l = l\,(y, r), \quad l_1(.) > 0, \quad l_2(.) < 0 \qquad (3)$$

where s is the desired private saving, y is the real GNP, θ is the (exogenously given) labour's share in GNP, t is the proportionate tax rate on labour earnings, i is the desired real physical investment, r is the interest rate and l is the desired real money holdings. The real money balance $\frac{M}{P}$, together with the real government spending g and the tax – rate t, are exogenous.

(i) The Laffer curve plots equilibrium tax collections on the vertical axis against t on the horizontal axis. Laffer's famous formula was that the curve slopes downwards. Show analytically whether or not that can happen here.

(ii) Suppose t is imposed on all factor payments, that is, on GNP. Reformulate equation (1) for this case. Does your answer to part (i) change?

Test code: ME I/ME II, 2005

Syllabus for ME I, 2005

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of a single variable, Theory of Sequence and Series**.**

Linear Programming: Formulations, statements of Primal and Dual problems. Graphical Solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2005

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

1. $X \sim B(n, p)$. The maximum value of Var (X) is

(A) $\frac{n}{4}$; (B) n ; (C) $\frac{n}{2}$; (D) $\frac{1}{n}$.

2. The function $x|x|$ is

(A) discontinuous at $x = 0$;

(B) continuous but not differentiable at $x = 0$;

(C) differentiable at $x = 0$;

(D) continuous everywhere but not differentiable anywhere.

3. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^5$ is

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; (B) $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$; (C) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$; (D) none of these.

4. The maximum and minimum values of the function

$f(x) = |x^2 + 2x - 3| + 1.5 \log_e x$, over the interval $\left[\frac{1}{2}, 4\right]$, are

(A) $(21 + 3 \log_e 2, -1.5 \log_e 2)$; (B) $(21 + \log_e 1.5, 0)$;

(C) $(21 + 3 \log_e 2, 0)$; (D) $(21 + \log_e 1.5, -1.5 \log_e 2)$.

5. Let α and β be the roots of the equation $x^2 - px + q = 0$. Define the sequence $x_n = \alpha^n + \beta^n$. Then x_{n+1} is given by

(A) $px_n - qx_{n-1}$; (B) $px_n + qx_{n-1}$;
(C) $qx_n - px_{n-1}$; (D) $qx_n + p\ x_{n-1}$.

6. Let $f : [-1,1] \to R$ be twice differentiable at $x = 0$, $f(0) = f'(0) = 0$, and $f''(0) = 4$. Then the value of $\underset{x\to 0}{Lt} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(A) 11 ; (B) 2 ; (C) 12; (D) none of these.

7. For $e < x_1 < x_2 < \infty$, $\dfrac{log_e x_2}{log_e x_1}$ is

(A) less than $\dfrac{x_2}{x_1}$; (B) greater than $\dfrac{x_2}{x_1}$, but less than $\left(\dfrac{x_2}{x_1}\right)^2$;

(C) greater than $\left(\dfrac{x_2}{x_1}\right)^3$; (D) greater than $\left(\dfrac{x_2}{x_1}\right)^2$, but less than $\left(\dfrac{x_2}{x_1}\right)^3$.

8. The value of the expression $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \dfrac{1}{\sqrt{99}+\sqrt{100}}$ is

(A) a rational number lying in the interval (0,9) ;
(B) an irrational number i lying in the interval (0,9) ;
(C) a rational number lying in the interval (0,10) ;
(D) an irrational number lying in the interval (0,10).

9. Consider a combination lock consisting of 3 buttons that can be pressed in any combination (including multiple buttons at a time), but in such a way that each number is pressed exactly once. Then the total number of possible combination locks with 3 buttons is

(A) 6 ; (B) 9 ; (C) 10 ; (D) 13 .

10. Suppose the correlation coefficient between x and y is denoted by R, and that between x and (y + x), by R_1.

Then, (A) $R_1 > R$; (B) $R_1 = R$; (C) $R_1 < R$ (D) none of these.

11. The value of $\int_{-1}^{1} (x + |x|)\, dx$ is

(A) 0; (B) -1; (C) 1; (D) none of these.

12. The values of $x_1 \geq 0$ and $x_2 \geq 0$ that maximize $\Pi = 45x_1 + 55x_2$ subject to $6x_1 + 4x_2 \leq 120$ and $3x_1 + 10x_2 \leq 180$ are

(A) (10,12) ; (B) (8,5) ; (C) (12,11) ; (D) none of the above.

Syllabus for ME II (Economics), 2005

Microeconomics: Theory of consumer behaviour, Theory of producer behaviour, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand) and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2005

1. (a) A divisible cake of size 1 is to be divided among n (>1) persons. It is claimed that the only allocation which is Pareto optimal allocation is $(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$. Do you agree with this claim ? Briefly justify your answer.

   (b) Which of the following transactions should be included in GDP? Explain whether the corresponding expenditure is a consumption expenditure or an investment expenditure.

   (i) Mr. Ramgopal , a private investment banker, hires Mr. Gopi to do cooking and cleaning at home.
   (ii) Mr. Ramgopal buys a new Maruti Esteem.
   (iii) Mr. Ramgopal flies to Kolkata from Delhi to see Durga Puja celebration.

(iv) Mr. Ramgopal directly buys (through the internet) 100 stocks of Satyam Ltd..

(v) Mr. Ramgopal builds a house.

2. Roses, once in full bloom, have to be picked up and sold on the same day. On any day the market demand function for roses is given by

$P = \alpha - Q$ (Q is number of roses ; P is price of a rose).

It is also given that the cost of growing roses, having been incurred by any owner of a rose garden long ago, is *not* a choice variable for him now.

(a) Suppose, there is only one seller in the market and he finds 1000 roses in full bloom on a day. How many roses should he sell on that day and at what price?

(b) Suppose there are 10 sellers in the market, and each finds in his garden 100 roses in full bloom ready for sale on a day. What will be the equilibrium price and the number of roses sold on that day? (To answer this part assume $\alpha \geq 1100$).

(c) Now suppose, the market is served by a large number of price taking sellers. However, the total availability on a day remains unchanged at 1000 roses. Find the competitive price and the total number of roses sold on that day.

3. Laxmi is a poor agricultural worker. Her consumption basket comprises three commodities: rice and two vegetables - *cabbage* and *potato*. But

there are occasionally very hard days when her income is so low that she can afford to buy only rice and no vegetables. However, there never arises a situation when she buys only vegetables and no rice. But when she can afford to buy vegetables, she buys only one vegetable, namely the one that has the lower price per kilogram on that day. Price of each vegetable fluctuates day to day while the price of rice is constant.

Write down a suitable utility function that would represent Laxmi's preference pattern. Explain your answer.

4. Consider a simple Keynesian model for a closed economy without Government. Suppose, saving is proportional to income (y), marginal propensity to invest with respect to y is 0.3 and the system is initially in equilibrium. Now, following a parallel downward shift of the saving function the equilibrium level of saving is found to increase by 12 units. Compute the change in the equilibrium income.

5. Consider an IS–LM model. In the commodity market let the consumption function be given by $C = a + b Y$, $a>0$, $0< b <1$. Investment and government spending are exogenous and given by I_0 and G_0 respectively. In the money market, the real demand for money is given by $L = kY - gr$, $k> 0$, $g >0$. The nominal money supply and price level are exogenously given at M_0 and P_0 respectively. In these relations C, Y and r denote consumption, real GDP and interest rate respectively.

(i) Set up the IS – LM equations.

(ii) Determine how an increase in the price level P_1, where $P_1 > P_0$, would affect real GDP and the interest rate.

Test code: ME I/ME II, 2006

Syllabus for ME I, 2006

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2006

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question.

1. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, $0 < x < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ equals

(A) $2f(x)$; (B) $\frac{f(x)}{2}$; (C) $(f(x))^2$; (D) none of these.

2. If $u = \phi(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ equals

(A) 0; (B) 1; (C) u; (D) none of these.

3. Let A and B be disjoint sets containing m and n elements, respectively, and let $C = A \cup B$. The number of subsets S of C that contain k elements and that also have the property that $S \cap A$ contains i elements is

(A) $\binom{m}{i}$; (B) $\binom{n}{i}$; (C) $\binom{m}{k-i}\binom{n}{i}$; (D) $\binom{m}{i}\binom{n}{k-i}$.

4. The number of disjoint intervals over which the function $f(x) = \left|0.5x^2 - |x|\right|$ is decreasing is

(A) one; (B) two; (C) three; (D) none of these.

5. For a set of real numbers $x_1, x_2, ..., x_n$, the root mean square (RMS) defined as RMS $= \left\{\frac{1}{N}\sum_{i=1}^{n} x_i^2\right\}^{1/2}$ is a measure of central tendency. If AM denotes the arithmetic mean of the set of numbers, then which of the following statements is correct?

(A) RMS < AM always; (B) RMS > AM always;
(C) RMS < AM when the numbers are not all equal;
(D) RMS > AM when numbers are not all equal.

6. Let $f(x)$ be a function of real variable and let Δf be the function $\Delta f(x) = f(x+1) - f(x)$. For $k > 1$, put $\Delta^k f = \Delta(\Delta^{k-1} f)$. Then $\Delta^k f(x)$ equals

(A) $\sum_{j=0}^{k}(-1)^j\binom{k}{j}f(x+j)$; (B) $\sum_{j=0}^{k}(-1)^{j+1}\binom{k}{j}f(x+j)$;
(C) $\sum_{j=0}^{k}(-1)^j\binom{k}{j}f(x+k-j)$; (D) $\sum_{j=0}^{k}(-1)^{j+1}\binom{k}{j}f(x+k-j)$.

7. Let $I_n = \int_0^{\infty} x^n e^{-x} dx$, where n is some positive integer. Then I_n equals

(A) $n! - nI_{n-1}$; (B) $n! + nI_{n-1}$; (C) nI_{n-1}; (D) none of these.

8. If $x^3 = 1$, then

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ equals}$$

(A) $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x & c & a \\ x^2 & a & b \end{vmatrix}$; (B) $(cx^2 + bx + a)\begin{vmatrix} x & b & c \\ 1 & c & a \\ x^2 & a & b \end{vmatrix}$;

(C) $(cx^2 + bx + a)\begin{vmatrix} x^2 & b & c \\ x & c & a \\ 1 & a & b \end{vmatrix}$; (D) $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$.

9. Consider any integer $I = m^2 + n^2$, where m and n are any two odd integers. Then

(A) I is never divisible by 2;
(B) I is never divisible by 4;
(C) I is never divisible by 6;
(D) none of these.

10. A box has 10 red balls and 5 black balls. A ball is selected from the box. If the ball is red, it is returned to the box. If the ball is black, it and 2 additional black balls are added to the box. The probability that a second ball selected from the box will be red is

(A) $\frac{47}{72}$; (B) $\frac{25}{72}$; (C) $\frac{55}{153}$; (D) $\frac{98}{153}$.

11. Let $f(x) = \dfrac{\log\left(1+\dfrac{x}{p}\right) - \log\left(1-\dfrac{x}{q}\right)}{x}$, $x \neq 0$. If f is continuous at $x = 0$, then the value of $f(0)$ is

(A) $\dfrac{1}{p} - \dfrac{1}{q}$; (B) $p + q$; (C) $\dfrac{1}{p} + \dfrac{1}{q}$; (D) none of these.

12. Consider four positive numbers x_1, x_2, y_1, y_2 such that $y_1\, y_2 > x_1\, x_2$. Consider the number $S = (x_1 y_2 + x_2 y_1) - 2x_1 x_2$. The number S is

(A) always a negative integer;
(B) can be a negative fraction;
(C) always a positive number;
(D) none of these.

13. Given $x \geq y \geq z$, and $x + y + z = 12$, the maximum value of $x + 3y + 5z$ is

(A) 36; (B) 42; (C) 38; (D) 32.

14. The number of positive pairs of integral values of (x, y) that solves $2xy - 4x^2 + 12x - 5y = 11$ is

(A) 4; (B) 1; (C) 2; (D) none of these.

15. Consider any continuous function f : [0, 1] $\rightarrow$ [0, 1]. Which one of the following statements is ***incorrect***?

(A) f always has at least one maximum in the interval [0, 1];
(B) f always has at least one minimum in the interval [0, 1];
(C) $\exists$ x $\in$ [0, 1] such that *f(x) = x;*
(D) the function f must always have the property that *f(0)* $\in$ *{0, 1},* *f(1)* $\in$ *{0, 1} and f(0) + f(1) =1.*

Syllabus for ME II (Economics), 2006

Microeconomics: Theory of consumer behaviour, Theory of production, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand (elementary problems)) and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model (with comparative statics), Harrod – Domar and Solow models, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2006

1.(a) There are two sectors producing the same commodity. Labour is perfectly mobile between these two sectors. Labour market is competitive and the representative firm in each of the two sectors maximizes profit. If there are 100 units of labour and the production function for sector i is: $F(L_i) = 15\sqrt{L_i}$, $i = 1,2$, find the allocation of labour between the two sectors.

(b) Suppose that prices of all variable factors and output double. What will be its effect on the short-run equilibrium output of a competitive firm? Examine whether the short-run profit of the firm will double.

(c) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	= Consumption	+ Investment	+ Export	- Import
1	____	____	____	____	____
2	____	____	____	____	____

2. A price-taking farmer produces a crop with labour L as the only input. His production function is: $F(L) = 10\sqrt{L} - 2L$. He has 4 units of labour

in his family and he cannot hire labour from the wage labour market. He does not face any cost of employing family labour.

(a) Find out his equilibrium level of output.

(b) Suppose that the government imposes an income tax at the rate of 10 per cent. How does this affect his equilibrium output?

(c) Suppose an alternative production technology given by: $F(L) = 11\sqrt{L} - L - 15$ is available. Will the farmer adopt this alternative technology? Briefly justify your answer.

3. Suppose a monopolist faces two types of consumers. In type *I* there is only one person whose demand for the product is given by : $Q_I = 100 - P$, where P represents price of the good. In type *II* there are n persons, each of whom has a demand for one unit of the good and each of them wants to pay a maximum of Rs. 5 for one unit. Monopolist cannot price discriminate between the two types. Assume that the cost of production for the good is zero. Does the equilibrium price depend on n ? Give reasons for your answer.

4. The utility function of a consumer is: $U(x, y) = xy$. Suppose income of the consumer (M) is 100 and the initial prices are $P_x = 5$, $P_y = 10$. Now suppose that P_x goes up to 10, P_y and M remaining unchanged. Assuming Slutsky compensation scheme, estimate price effect, income effect and substitution effect.

5. Consider an *IS-LM* model for a closed economy. Private consumption depends on disposable income. Income taxes (T) are lump-sum. Both private investment and speculative demand for money vary inversely with interest rate (r). However, transaction demand for money depends not on income (y) but on disposable income (y_d). Argue how the equilibrium values of private investment, private saving, government saving, disposable income and income will change, if the government raises T.

6. An individual enjoys bus ride. However, buses emit smoke which he dislikes. The individual's utility function is: $U = U(x, s)$, where x is the distance (in km) traveled by bus and s is the amount of smoke consumed from bus travel.

(a) What could be the plausible alternative shapes of indifference curve between x and s?

(b) Suppose, smoke consumed from bus travel is proportional to the distance traveled: $s = \alpha x$ (α is a positive parameter). Suppose further that the bus fare per km is p and that the individual has money income M to spend on bus travel. Show the budget set of the consumer in an (s, x) diagram.

(c) What can you say about an optimal choice of the individual? Will he necessarily exhaust his entire income on bus travel?

7. (a) Suppose the labour supply (l) of a household is governed by maximization of its utility (u): $u = c^{2/3}h^{1/3}$, where c is the household's consumption and h is leisure enjoyed by the household (with $h + l = 24$). Real wage rate (w) is given and the household consumes the entire labour income (wl). What is the household's labour supply? Does it depend on w?

(b) Consider now a typical Keynesian (closed) economy producing a single good and having a single household. There are two types of final expenditure – viz., investment autonomously given at 36 units and household consumption (c) equalling the household's labour income (wl). It is given that $w = 4$. Firms produce aggregate output (y) according to the production function: $y = 24\sqrt{l}$. Find the equilibrium level of output and employment. Is there any involuntary unemployment? If so, how much?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\,\frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

iv) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

v) Give an intuitive explanation of your finding in (iv)

9. A consumer consumes only two commodities x_1 and x_2. Suppose that her utility function is given by $U(x_1, x_2) = \min\ (2x_1, x_2)$.

(i) Draw a representative indifference curve of the consumer.

(ii) Suppose the prices of the commodities are Rs.5 and Rs.10 respectively while the consumer's income is Rs. 100. What commodity bundle will the consumer purchase?

(iii) Suppose the price of commodity 1 now increases to Rs. 8. Decompose the change in the amount of commodity 1 purchased into income and substitution effects.

10. A price taking firm makes machine tools Y using labour and capital according to the production function Y = $K^{0.25}L^{0.25}$. Labour can be hired at the beginning of every week while capital can be hired only at the beginning of every month. Let one month be considered as long run period and one week as short run period. Further assume that one month equals four weeks. The wage rate per week and the rental rate of capital per month are both 10.

(i) Given the above information, find the short run and the long run cost functions of the firm.

(ii) At the beginning of the month of January, the firm is making long run decisions given that the price of machine tools is 400. What is the long run profit maximizing number of machine tools? How many units of labour and capital should the firm hire at the beginning of January?

11. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

Test code: ME I/ME II, 2007

Syllabus for ME I, 2007

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2007

1. Let α and β be any two positive real numbers. Then

   $Lim_{x\to 0} \dfrac{(1+x)^{\alpha}-1}{(1+x)^{\beta}-1}$ equals

   (A) $\dfrac{\alpha}{\beta}$; (B) $\dfrac{\alpha+1}{\beta+1}$; (C) $\dfrac{\alpha-1}{\beta-1}$; (D) 1.

2. Suppose the number X is odd. Then X^2-1 is

   (A) odd; (B) not prime;

   (C) necessarily positive; (D) none of the above.

3. The value of k for which the function $f(x) = ke^{kx}$ is a probability density function on the interval [0, 1] is

(A) $k = \log 2$; (B) $k = 2\log 2$; (C) $k = 3\log 3$; (D) $k = 3\log 4$.

4. p and q are positive integers such that $p^2 - q^2$ is a prime number. Then, $p - q$ is

(A) a prime number; (B) an even number greater than 2;

(C) an odd number greater than 1 but not prime; (D) none of these.

5. Any non-decreasing function defined on the interval $[a, b]$

(A) is differentiable on (a, b);

(B) is continuous in $[a, b]$ but not differentiable;

(C) has a continuous inverse;

(D) none of these.

6. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 0$ is satisfied by

(A) $x = 1$; (B) $x = 3$; (C) $x = 4$; (D) none of these.

7. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}}$, then $f'(x)$ is

(A) $\dfrac{x}{2f(x) - 1}$; (B) $\dfrac{1}{2f(x) - 1}$; (C) $\dfrac{1}{x\sqrt{f(x)}}$; (D) $\dfrac{1}{2f(x) + 1}$.

8. If $P = \log_x(xy)$ and $Q = \log_y(xy)$, then $P + Q$ equals

(A) PQ; (B) P/Q; (C) Q/P; (D) $(PQ)/2$.

9. The solution to $\int \dfrac{2x^3 + 1}{x^4 + 2x}\, dx$ is

(A) $\dfrac{x^4 + 2x}{4x^3 + 2}$ + constant; (B) $\log x^4 + \log 2x$ + constant;

(C) $\frac{1}{2}\log|x^4+2x|$ + constant; (D) $\left|\frac{x^4+2x}{4x^3+2}\right|$ + constant.

10. The set of all values of x for which $x^2-3x+2>0$ is

(A) $(-\infty,1)$; (B) $(2,\infty)$; (C) $(-\infty,2)\cap(1,\infty)$; (D) $(-\infty,1)\cup(2,\infty)$.

11. Consider the functions $f_1(x)=x^2$ and $f_2(x)=4x^3+7$ defined on the real line. Then

(A) f_1 is one-to-one and onto, but not f_2;

(B) f_2 is one-to-one and onto, but not f_1;

(C) both f_1 and f_2 are one-to-one and onto;

(D) none of the above.

12. If $f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2x}$, $a>0,\ b>0$, then $f'(0)$ equals

(A) $\left(\frac{b^2-a^2}{b^2}\right)\left(\frac{a}{b}\right)^{a+b-1}$; (B) $\left(2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$;

(C) $2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}$; (D) $\left(\frac{b^2-a^2}{ba}\right)$.

13. The linear programming problem

$$\max_{x,y}\ z=0.5x+1.5y$$

subject to: $x+y\le 6$

$3x+y\le 15$

$x+3y\le 15$

$x,y\ge 0$

has

(A) no solution; (B) a unique non-degenerate solution;

(C) a corner solution; (D) infinitely many solutions.

14. Let $f(x;\,\theta) = \theta f(x;1) + (1-\theta) f(x;0)$, where θ is a constant satisfying $0 < \theta < 1$. Further, both $f(x;1)$ and $f(x;0)$ are probability density functions (*p.d.f.*). Then

(A) $f(x;\theta)$ is a *p.d.f.* for all values of θ;

(B) $f(x;\theta)$ is a *p.d.f.* only for $0 < \theta < \frac{1}{2}$;

(C) $f(x;\theta)$ is a *p.d.f.* only for $\frac{1}{2} \le \theta < 1$;

(D) $f(x;\theta)$ is not a *p.d.f.* for any value of θ.

15. The correlation coefficient *r* for the following five pairs of observations

x	5	1	4	3	2
y	0	4	2	0	-1

satisfies

(A) $r > 0$; (B) $r < -0.5$; (C) $-0.5 < r < 0$; (D) $r = 0$.

16. An n-coordinated function f is called homothetic if it can be expressed as an increasing transformation of a homogeneous function of degree one. Let $f_1(x) = \sum_{i=1}^{n} x_i^r$, and $f_2(x) = \sum_{i=1}^{n} a_i x_i + b$, where $x_i > 0$ for all *i*, $0 < r < 1$, $a_i > 0$ and *b* are constants. Then

(A) f_1 is not homothetic but f_2 is; (B) f_2 is not homothetic but f_1 is;

(C) both f_1 and f_2 are homothetic; (D) none of the above.

17. If $h(x) = \frac{1}{1-x}$, then $h(h(h(x)))$ equals

(A) $\frac{1}{1-x}$; (B) x; (C) $\frac{1}{x}$; (D) 1- x.

18. The function $x|x|+\left(\frac{|x|}{x}\right)^3$ is

(A) continuous but not differentiable at $x = 0$;

(B) differentiable at $x = 0$;

(C) not continuous at $x = 0$;

(D) continuously differentiable at $x = 0$.

19. $\int \frac{2dx}{(x-2)(x-1)x}$ equals

(A) $\log\left|\frac{x(x-2)}{(x-1)^2}\right|$ + constant;

(B) $\log\left|\frac{(x-2)}{x(x-1)^2}\right|$ + constant;

(C) $\log\left|\frac{x^2}{(x-1)(x-2)}\right|$ + constant;

(D) $\log\left|\frac{(x-2)^2}{x(x-1)}\right|$ + constant.

20. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

(A) $1-\frac{209}{552}$; (B) $1-14\times\left(\frac{4}{5}\right)^{52}$; (C) $\left(\frac{4}{5}\right)^{50}$; (D) $\left(\frac{1}{5}\right)^{50}$.

21. For any real number *x,* define [*x*] as the highest integer value not greater than *x*. For example, [0.5] = 0, [1] = 1 and [1.5] = 1. Let $I = \int_0^{3/2} \{[x]+[x^2]\} dx$. Then *I* equals

(A) 1; (B) $\frac{5-2\sqrt{2}}{2}$;

(C) $2\sqrt{2}$; (D) none of these.

22. Every integer of the form $(n^3 - n)(n^2 - 4)$ (for $n = 3, 4, \ldots$) is

(A) divisible by 6 but not always divisible by 12;

(B) divisible by 12 but not always divisible by 24;

(C) divisible by 24 but not always divisible by 120;

(D) divisible by 120 but not always divisible by 720.

23. Two varieties of mango, A and B, are available at prices Rs. p_1 and Rs. p_2 per kg, respectively. One buyer buys 5 kg. of A and 10 kg. of B and another buyer spends Rs. 100 on A and Rs. 150 on B. If the average expenditure per mango (irrespective of variety) is the same for the two buyers, then which of the following statements is the most appropriate?

(A) $p_1 = p_2$; (B) $p_2 = \frac{3}{4} p_1$;

(C) $p_1 = p_2$ or $p_2 = \frac{3}{4} p_1$; (D) $\frac{3}{4} \le \frac{p_2}{p_1} < 1$.

24. For a given bivariate data set $(x_i, y_i;\ i = 1, 2, \ldots, n)$, the squared correlation coefficient (r^2) between x^2 and *y* is found to be 1. Which of the following statements is the most appropriate?

(A) In the (x, y) scatter diagram, all points lie on a straight line.

(B) In the (x, y) scatter diagram, all points lie on the curve $y = x^2$.

(C) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2$, $a > 0$, $b > 0$.

(D) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2$, a, b any real numbers.

25. The number of possible permutations of the integers 1 to 7 such that the numbers 1 and 2 always precede the number 3 and the numbers 6 and 7 always succeed the number 3 is

(A) 720; (B) 168;

(C) 84; (D) none of these.

26. Suppose the real valued continuous function f defined on the set of non-negative real numbers satisfies the condition $f(x) = xf(x)$, then $f(2)$ equals

(A) 1; (B) 2;

(C) 3; (D) $f(1)$.

27. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$;

(B) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$;

(C) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$;

(D) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

28. Consider a square that has sides of length 2 units. *Five* points are placed anywhere inside this square. Which of the following statements is **incorrect**?

(A) There cannot be any two points whose distance is more than $2\sqrt{2}$.

(B) The square can be partitioned into four squares of side 1 unit each such that at least one unit square has two points that lies on or inside it.

(C) At least two points can be found whose distance is less than $\sqrt{2}$.

(D) Statements (A), (B) and (C) are all incorrect.

29. Given that f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

(A) $f(x)$ is an increasing function;

(B) $f(x)$ is a decreasing function;

(C) $|f(x)|$ is an increasing function;

(D) $|f(x)|$ is a decreasing function.

30. Let p, q, r, s be four arbitrary positive numbers. Then the value of $\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ is at least as large as

(A) 81;

(B) 91;

(C) 101.

(D) None of these.

Syllabus for ME II (Economics), 2007

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2007

1. (a) There is a cake of size 1 to be divided between two persons, 1 and 2. Person 1 is going to cut the cake into two pieces, but person 2 will select one of the two pieces for himself first. The remaining piece will go to person 1. What is the optimal cutting decision for player 1? Justify your answer.

(b) Kamal has been given a free ticket to attend a classical music concert. If Kamal had to pay for the ticket, he would have paid up to Rs. 300/- to attend the concert. On the same evening, Kamal's alternative entertainment option is a film music and dance event for which tickets are priced at Rs. 200/- each. Suppose also that Kamal is willing to pay up to Rs. X to attend the film music and dance event. What does Kamal do, i.e., does he attend the classical music concert, or does he attend the film music and dance show, or does he do neither? Justify your answer.

2. Suppose market demand is described by the equation P = 300 – Q and competitive conditions prevail. The short-run supply curve is P = -180 +5Q. Find the initial short-run equilibrium price and quantity. Let the long-run supply curve be P = 60 + 2Q. Verify whether the market is also in the long-run equilibrium at the initial short-run equilibrium that you have worked out. Now suppose that the market demand at every price is

doubled. What is the new market demand curve? What happens to the equilibrium in the *very* short-run? What is the new short-run equilibrium? What is the new long-run equilibrium? If a price ceiling is imposed at the old equilibrium, estimate the perceived shortage. Show all your results in a diagram.

3. (a) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	=	Consumption	+	Investment	+	Export	-	Import
1	____		____		____		____		____
2	____		____		____		____		____

(b) Consider an IS-LM model for a closed economy with government where investment (I) is a function of rate of interest (r) only. An increase in government expenditure is found to crowd out 50 units of private investment. The government wants to prevent this by a minimum change in the supply of real money balance. It is given that $\frac{dI}{dr} = -50$, slope of the LM curve, $\frac{dr}{dy}(LM) = \frac{1}{250}$, slope of the IS curve, $\frac{dr}{dy}(IS) = -\frac{1}{125}$, and all relations are linear. Compute the change in y from the initial to the final equilibrium when all adjustments have been made.

4. (a) Consider a consumer with income W who consumes *three* goods, which we denote as $i = 1, 2, 3$. Let the amount of good i that the consumer consumes be x_i and the price

of good i be p_i. Suppose that the consumer's preference is described by the utility function $U(x_1, x_2, x_3) = x_1\sqrt{x_2 x_3}$.

(i) Set up the utility maximization problem and write down the Lagrangian.

(ii) Write down the first order necessary conditions for an interior maximum and then obtain the Marshallian (or uncompensated) demand functions.

(b) The production function, $Y = F(K, L)$, satisfies the following properties: (i) CRS, (ii) symmetric in terms of inputs and (iii) $F(1,1) = 1$. The price of each input is Rs. 2/- per unit and the price of the product is Rs. 3/- per unit. *Without using calculus* find the firm's optimal level of production.

5.(a) A monopolist has contracted to sell as much of his output as he likes to the government at Rs.100/- per unit. His sale to the government is positive. He also sells to private buyers at Rs 150/- per unit. What is the price elasticity of demand for the monopolist's products in the private market?

(b) Mrs. Pathak is very particular about her consumption of tea. She always takes 50 grams of sugar with 20 grams of ground tea. She has allocated Rs 55 for her spending on tea and sugar per month. (Assume that she doesn't offer tea to her guests or anybody else and she doesn't consume sugar for any other purpose). Sugar and tea are sold at 2 paisa per 10 grams and 50 paisa per 10 grams respectively. Determine how much of tea and sugar she demands per month.

(c) Consider the IS-LM model with government expenditure and taxation. A change in the income tax rate changes the equilibrium from $(y = 3000,\ r = 4\%)$ to $(y = 3500,\ r = 6\%)$, where y, r denote income and rate of interest, respectively. It is given that a unit increase in y increases demand for real money balance by 0.25 of a unit. Compute the change in real money demand that results from a 1% increase in the rate of interest. (Assume that all relationships are linear**.**)

6. (a) An economy produces two goods, corn and machine, using for their production only labor and some of the goods themselves. Production of one unit of corn requires 0.1 units of corn, 0.3 machines and 5 man-hours of labor. Similarly, production of one machine requires 0.4 units of corn, 0.6 machines and 20 man-hours of labor.

(i) If the economy requires 48 units of corn but no machine for final consumption, how much of each of the two commodities is to be produced? How much labor will be required?

(ii) If the wage rate is Rs. 2/- per man-hour, what are the prices of corn and machines, if price of each commodity is equated to its average cost of production?

(b) Consider two consumers *A* and *B*, each with income *W*. They spend their entire budget over the two commodities, X and Y. Compare the demand curves of the two consumers under the assumption that their utility functions are $U_A = x + y$ and $U_B = x^2 + y^2$ respectively.

7. Consider a Simple Keynesian Model without government for an open economy, where both consumption and import are proportional functions of income (Y). Suppose that average propensities to consume and import are 0.8 and 0.3, respectively. The investment (I) function and the level of export (X) are given by $I = 100 + 0.4Y$ and $X = 100$.

(i) Compute the aggregate demand function if the maximum possible level of imports is 450. Can there be an equilibrium for this model? Show your result graphically.

(ii) How does your answer to part (i) change if the limit to import is raised to 615? What can you say about the stability of equilibrium if it exists?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

(iv) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

(v) Give an intuitive explanation of your finding in (iv)

9. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

Test code: ME I/ME II, 2008

Syllabus for ME I, 2008

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).
Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2008

1. $\int \frac{dx}{x + x\log x}$ equals

(a) $\log|x + x\log x| +$ constant

(b) $\log|1 + x\log x| +$ constant

(c) $\log|\log x| +$ constant

(d) $\log|1 + \log x| +$ constant.

2. The inverse of the function $\sqrt{-1+x}$ is

(a) $\frac{1}{\sqrt{x-1}}$, (b) $x^2 + 1$, (c) $\sqrt{x-1}$, (d) none of these.

3. The domain of continuity of the function $f(x) = \sqrt{x} + \frac{x+1}{x-1} - \frac{x+1}{x^2+1}$ is

(a) $[0,1)$, (b) $(1,\infty)$, (c) $[0,1) \cup (1,\infty)$, (d) none of these

4. Consider the following linear programme:

minimise $x - 2y$

subject to $x + 3y \geq 3$

$3x + y \geq 3$

$x + y \leq 3$

An optimal solution of the above programme is given by

(a) $x = \frac{3}{4}, \quad y = \frac{3}{4}$.

(b) $x = 0, \quad y = 3$

(c) $x = -1, y = 3$.

(d) none of (a), (b) and (c).

5. Consider two functions $f_1 : \{a_1, a_2, a_3\} \to \{b_1, b_2, b_3, b_4\}$ and $f_2 : \{b_1, b_2, b_3, b_4\} \to \{c_1, c_2, c_3\}$. The function f_1 is defined by $f_1(a_1) = b_1$, $f_1(a_2) = b_2$, $f_1(a_3) = b_3$ and the function f_2 is defined by $f_2(b_1) = c_1$, $f_2(b_2) = c_2$, $f_2(b_3) = f_2(b_4) = c_3$. Then the mapping $f_2 \circ f_1 : \{a_1, a_2, a_3\} \to \{c_1, c_2, c_3\}$ is

(a) a composite and one – to – one function but not an onto function.
(b) a composite and onto function but not a one – to – one function.
(c) a composite, one – to – one and onto function.
(d) not a function.

6. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$, $t > 0$, $t \neq 1$ then the relation between x and y is

(a) $y^x = x^{\frac{1}{y}}$, (b) $x^{\frac{1}{y}} = y^{\frac{1}{x}}$, (c) $x^y = y^x$, (d) $x^y = y^{\frac{1}{x}}$.

7. The maximum value of $T = 2x_B + 3x_S$ subject to the constraint $20x_B + 15x_S \leq 900$, where $x_B \geq 0$ and $x_S \geq 0$, is

(a) 150, (b) 180, (c) 200, (d) none of these.

8. The value of $\int_0^2 [x]^n f'(x)\,dx$, where $[x]$ stands for the integral part of x, n is a positive integer and f' is the derivative of the function f, is

(a) $(n+2^n)(f(2)-f(0))$, (b) $(1+2^n)(f(2)-f(1))$,
(c) $2^n f(2)-(2^n-1)f(1)-f(0)$, (d) none of these.

9. A surveyor found that in a society of 10,000 adult literates 21% completed college education, 42% completed university education and remaining 37% completed only school education. Of those who went to college 61% reads newspapers regularly, 35% of those who went to the university and 70% of those who completed only school education are regular readers of newspapers. Then the percentage of those who read newspapers regularly completed only school education is

(a) 40%, (b) 52%, (c) 35%, (d) none of these.

10. The function $f(x) = x|x|e^{-x}$ defined on the real line is
(a) continuous but not differentiable at zero,
(b) differentiable only at zero,
(c) differentiable everywhere,
(d) differentiable only at finitely many points.

11. Let X be the set of positive integers denoting the number of tries it takes the Indian cricket team to win the World Cup. The team has equal odds for winning or losing any match. What is the probability that they will win in odd number of matches?

(a) 1/4, (b) 1/2, (c) 2/3, (d) 3/4.

12. Three persons X, Y, Z were asked to find the mean of 5000 numbers, of which 500 are unities. Each one did his own simplification.

$X's$ method: Divide the set of number into 5 equal parts, calculate the mean for each part and then take the mean of these.
$Y's$ method: Divide the set into 2000 and 3000 numbers and follow the procedure of A.
$Z's$ method: Calculate the mean of 4500 numbers (which are $\neq 1$) and then add 1.
Then

(a) all methods are correct,
(b) $X's$ method is correct, but Y and $Z's$ methods are wrong,
(c) $X's$ and $Y's$ methods are correct but $Z's$ methods is wrong,
(d) none is correct.

13. The number of ways in which six letters can be placed in six directed envelopes such that exactly four letters are placed in correct envelopes and exactly two letters are placed in wrong envelopes is
(a) 1, (b) 15, (c) 135. (d) None of these.

14. The set of all values of x for which the inequality $|x-3|+|x+2|<11$ holds is
(a) (– 3, 2), (b) (– 5, 2), (c) (–5, 6), (d) none of these.

15. The function $f(x)=x^4-4x^3+16x$ has
(a) a unique maximum but no minimum,
(b) a unique minimum but no maximum,
(c) a unique maximum and a unique minimum,
(d) neither a maximum nor a minimum.

16. Consider the number $K(n)=(n+3)(n^2+6n+8)$ defined for integers n. Which of the following statements is correct?
(a) *K (n)* is always divisible by 4,
(b) *K (n)* is always divisible by 5,
(c) *K (n)* is always divisible by 6,
(d) Statements (a), (b) and(c) are incorrect.

17. 25 books are placed at random on a shelf. The probability that a particular pair of books shall be always together is
(a) $\frac{2}{25}$, (b) $\frac{1}{25}$, (c) $\frac{1}{300}$, (d) $\frac{1}{600}$.

18. $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If -1 is a root of the equation, the other root is

(a) $\frac{4}{5}$, (b) $\frac{8}{5}$, (c) $\frac{6}{5}$, (d) $\frac{3}{5}$.

19. The correlation coefficients between two variables X and Y obtained from the two equations $2x + 3y - 1 = 0$ and $5x - 2y + 3 = 0$ are

(a) equal but have opposite signs,

(b) $-\frac{2}{3}$ and $\frac{2}{5}$,

(c) $\frac{1}{2}$ and $-\frac{3}{5}$,

(d) Cannot say.

20. If a, b, c, d are positive real numbers then $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$ is always

(a) less than $\sqrt{2}$,

(b) less than 2 but greater than or equal to $\sqrt{2}$,

(c) less than 4 but greater than or equal 2,

(d) greater than or equal to 4.

21. The range of value of x for which the inequality $\log_{(2-x)}(x-3) \geq -1$ holds is

(a) $2 < x < 3$, (b) $x > 3$, (c) $x < 2$, (d) no such x exists.

22. The equation $5x^3 - 5x^2 + 2x - 1$ has

(a) all roots between 1 and 2,

(b) all negative roots,

(c) a root between 0 and 1,

(d) all roots greater than 2.

23. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \qquad (k > 0, 0 \leq x \leq \infty)$$

Then, a equals

(a) $\frac{k^3}{2}$, (b) $\frac{k}{2}$, (c) $\frac{k^2}{2}$, (d) k.

24. Let $x = r$ be the mode of the distribution with probability mass function $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Then which of the following inequalities hold.

(a) $(n+1)p - 1 < r < (n+1)p$,

(b) $r < (n+1)p - 1$,

(c) $r > (n+1)p$,

(d) $r < np$.

25. Let $y = (y_1, \ldots, y_n)$ be a set of n observations with $y_1 \le y_2 \le \ldots \le y_n$. Let $y' = (y_1, y_2, \ldots, y_j + \delta, \ldots, y_k - \delta, \ldots, y_n)$ where $y_k - \delta > y_{k-1} > \ldots. > y_{j+1} > y_j + \delta$, $\delta > 0$. Let σ : standard deviation of y and σ' : standard deviation of y'. Then

(a) $\sigma < \sigma'$, (b) $\sigma' < \sigma$, (c) $\sigma' = \sigma$, (d) nothing can be said.

26. Let x be *a r.v.* with *pdf* $f(x)$ and let $F(x)$ be the distribution function. Let $r(x) = \dfrac{xf(x)}{1 - F(x)}$. Then for $x < e^{\mu}$ and $f(x) = \dfrac{e^{-\frac{(\log x - \mu)^2}{2}}}{x\sqrt{2\pi}}$, the function $r(x)$ is

(a) increasing in x,

(b) decreasing in x,

(c) constant,

(d) none of the above.

27. A square matrix of order n is said to be a bistochastic matrix if all of its entries are non-negative and each of its rows and columns sum to 1. Let $y_{n\times 1} = P_{n\times n} . x_{n\times 1}$ where elements of y are some rearrangements of the elements of x. Then

(a) P is bistochastic with diagonal elements 1,

(b) P cannot be bistochastic,

(c) P is bistochastic with elements 0 and 1,

(d) P is a unit matrix.

28. Let $f_1(x) = \dfrac{x}{x+1}$. Define $f_n(x) = f_1(f_{n-1}(x))$, where $n \ge 2$. Then $f_n(x)$ is

(a) decreasing in n, (b) increasing in n, (c) initially decreasing in n and then increasing in n, (d) initially increasing in n and then decreasing n.

29. $\lim\limits_{n\to\infty} \dfrac{1 - x^{-2n}}{1 + x^{-2n}}$, $x > 0$ equals

(a) 1, (b) - 1, (c) 0. (d) The limit does not exist.

30. Consider the function $f(x_1, x_2) = \max\{6 - x_1, 7 - x_2\}$. The solution (x_1^*, x_2^*) to the optimization problem minimize $f(x_1, x_2)$ subject to $x_1 + x_2 = 21$ is

(a) $(x_1^* = 10.5,\ x_2^* = 10.5)$,

(b) $(x_1^* = 11,\ x_2^* = 10)$,

(c) $(x_1^* = 10,\ x_2^* = 11)$,

(d) None of these.

Syllabus for ME II (Economics), 2008

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2008

1. There are two individuals A and B and two goods *X* and *Y*. The utility functions of A and B are given by $U_A = X_A$ and $U_B = X_B^2 + Y_B^2$ respectively where X_i, Y_i are consumption levels of the two goods by individual *i*, *i* = *A*, *B*.

    (a) Draw the indifference curves of A and B.
    (b) Suppose A is endowed with 10 units of Y and B with 10 units of X. Indicate the endowment point in a box diagram.
    (c) Draw the set of Pareto optimal allocation points in the box diagram.

2. Suppose an economy's aggregate output (*Y*) is given by the following production function:

$$Y = U N^{\alpha}, \ (0 < \alpha < 1)$$

where *U*, a random variable, represents supply shock. Employment of labour (*N*) is determined by equating its marginal product to $\frac{W}{P}$, where *W* is nominal wage and *P* is price level.

Use the notations: $u = \log \alpha + \frac{1}{\alpha} \log U$; *p* = log *P*; *w* = log *W* and *y* = log *Y*.

(a) Obtain the aggregate supply function (*y*) in terms of *p*, *w*, and *u*.
(b) Add the following relations:

Wages are indexed: $w = \theta p, \ (0 \le \theta \le 1)$
Aggregate demand: *y* = *m* – *p*, (*m* = logarithm of money, a policy variable)

Find the solution of *y* in terms of *m* and *u*.

(c) Does monetary policy affect output (i) if indexation is partial $(0 < \theta < 1)$, (ii) indexation is full $(\theta = 1)$?
(d) Does the real shock affect output more when indexation is higher? Explain.

3. Two firms 1 and 2 sell a single, homogeneous, infinitely divisible good in a market. Firm 1 has 40 units to sell and firm 2 has 80 units to sell. Neither firm can produce any more units. There is a demand curve: $p = a - q$, where *q* is the total amount placed by the firms in the market. So if q_i is the amount placed by firm *i*, $q = q_1 + q_2$ and *p* is the price that emerges. *a* is positive and a measure of market size. It is known that *a* is either 100 or 200. The value of *a* is observed by both firms. After they observe the value of *a*, each firm decides whether or not to destroy a part of its output. This decision is made simultaneously and independently by the firms. Each firm faces a constant per unit cost of destruction

equal to 10. Whatever number of units is left over after destruction is sold by the firm in the market.

Show that a firm's choice about the amount it wishes to destroy is independent of the amount chosen by the other firm. Show also that the amount destroyed by firm 2 is always positive, while firm 1 destroys a part of its output if and only if a = 100.

4. (a) Two commodities, X and Y, are produced with identical technology and are sold in competitive markets. One unit of labour can produce one unit of each of the two commodities. Labour is the only factor of production; and labour is perfectly mobile between the two sectors. The representative consumer has the utility function: $U = \sqrt{XY}$; and his income is Rs. 100/-. If 10 units of labour are available, find out the equilibrium wage in the competitive labour market.

(b) Consider an economy producing a single good by a production function

$$Y = \min\{K, L\}$$

where Y is the output of the final good. K and L are input use of capital and labour respectively. Suppose this economy is endowed with 100 units of capital and labour supply L_s is given by the function

$$L_s = 50\,w\,,$$

where w is the wage rate.

Assuming that all markets are competitive find the equilibrium wage and rental rate.

5. The following symbols are used: Y = output, N = employment, W = nominal wage, P = price level, P^e = expected price level.

The Lucas supply function is usually written as:

$$\log Y = \log Y^* + \lambda \ (\log P - \log P^e)$$

where Y^* is the natural level of output. Consider an economy in which labour supply depends positively on the expected real wage:

$$\frac{W}{P^e} = N^\sigma,\ (\sigma > 0) \quad \text{(labour supply)}$$

Firms demand labour up to the point where its marginal product equals the given (actual) real wage $\left(\frac{W}{P}\right)$ and firm's production function is:

$$Y = N^\alpha,\ (0 < \alpha < 1)$$

(a) Find the labour demand function.
(b) Equate labour demand with labour supply to eliminate W. You will get an expression involving P, P^e and N. Derive the Lucas supply function in the form given above and find the expressions for λ and Y^*.
(c) How is this type of model referred to in the literature? Explain

6. Consider an IS – LM model given by the following equations

$$C = 200 + .5\, Y_D$$
$$I = 150 - 1000\, r$$
$$T = 200$$
$$G = 250$$
$$\left(\frac{M}{P}\right)^d = 2Y - 4000\, i$$
$$\left(\frac{M}{P}\right)^s = 1600$$
$$i = r - \Pi^e$$

where C is consumption, Y_D is disposable income, I is investment, r is real rate of interest, i is nominal rate of interest, T is tax, G is government expenditure, $\left(\frac{M}{P}\right)^d$ and $\left(\frac{M}{P}\right)^s$ are real money demand and real money supply respectively and and Π^e is the expected rate of inflation. The current price level P remains always rigid.

(a) Assuming that $\Pi^e = 0$, i.e., the price level is expected to remain unchanged in future, determine the equilibrium levels of income and the rates of interest.
(b) Suppose there is a *temporary* increase in nominal money supply by 2%. Find the new equilibrium income and the rates of interest.
(c) Now assume that the 2% increase in nominal money supply is *permanent* leading to a 2% increase in the expected future price level. Work out the new equilibrium income and the rates of interest.

7. A firm is contemplating to hire a salesman who would be entrusted with the task of selling a washing machine. The hired salesman is efficient with probability 0.25 and inefficient with probability 0.75 and there is no way to tell, by looking at the salesman, if he is efficient or not. An efficient salesman can sell the washing machine with probability 0.8 and an inefficient salesman can sell the machine with probability 0.4. The firm makes

a profit of Rs. 1000 if the machine is sold and gets nothing if it is not sold. In either case, however, the salesman has to be paid a wage of Rs. 100.

(a) Calculate the expected profit of the firm.
(b) Suppose instead of a fixed payment, the firm pays a commission of t % on its profit to the salesman (i.e., if the good is sold the salesman gets Rs. $1000 \times \frac{t}{100}$ and nothing if the good remains unsold). A salesman, irrespective of whether he is efficient or inefficient, has an alternative option of working for Rs. 80. A salesman knows whether he is efficient or not and cares only about the expected value of his income: find the value of t that will maximize the expected profit of the firm.

8. (a) On a tropical island there are 100 boat builders, numbered 1 through 100. Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i, from 1 to 100, boat builder has a cost function $C_i(y) = 11 + i\,y$. Assume that in the cost function the fixed cost, 11, is a quasi-fixed cost, that is, it is only paid if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

(b) Consider the market for a particular good. There are two types of customers: those of type 1 are the low demand customers, each with a demand function of the form $p = 10 - q_1$, and those of type 2, who are the high demand customers, each with a demand function of the form $p = 2(10 - q_2)$. The firm producing the product is a monopolist in this market and has a cost function $C(q) = 4q^2$ where $q = q_1 + q_2$.

(i) Suppose the firm is unable to prevent the customers from selling the good to one another, so that the monopolist cannot charge different customers different prices. What prices per unit will the monopolist charge to maximize its total profit and what will be the equilibrium quantities to be supplied to the two groups in equilibrium?

(ii) Suppose the firm realizes that by asking for IDs it can identify the types of the customers (for instance, type 1's are students who can be identified using their student IDs). It can thus charge different per unit prices to the two groups, if it is optimal to do so. Find the profit maximizing prices to be charged to the two groups.

9. Consider the following box with 16 squares:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

There are two players 1 and 2, and the game begins with player 1 selecting one of the boxes marked 1 to 16. Following such a selection, the selected box, as well as all boxes in the square of which the selected box constitutes the *leftmost and lowest* corner, will be deleted. For example, if he selects box 7, then all the boxes, 3, 4, 7 and 8 are deleted. Similarly, if he selects box 9, then all boxes 1 to 12 are deleted. Next it is player 2's turn to select a box from the remaining boxes. The same deletion rule applies in this case. It is then player 1's turn again, and so on. Whoever deletes the last box loses the game. What is a winning strategy for player 1 in this game?

10. (i) Mr. *A's* yearly budget for his car is Rs. 100,000, which he spends completely on petrol (*P*) and on all other expenses for his car (*M*). All other expenses for car (*M*) is measured in Rupees, so you can consider that price of *M* is Re. 1. When price of petrol is Rs. 40 per liter, Mr. A buys 1,000 liters per year.
The petrol price rises to Rs. 50 per liter, and to offset the harm to Mr. A, the government gives him a cash transfer of Rs. 10,000 per year.

(a) Write down Mr. A's yearly budget equation under the 'price rise plus transfer' situation.
(b) What will happen to his petrol consumption – increase, decrease, or remain the same?
(c) Will he be better or worse off after the price rise plus transfer than he was before?

[Do not refer to any utility function or indifference curves to answer (b) and (c)]

(iii) Mr. B earns Rs. 500 today and Rs. 500 tomorrow. He can save for future by investing today in bonds that return tomorrow the principal plus the interest. He can also borrow from his bank paying an interest. When the interest rates on both bank loans and bonds are 15% Mr. B chooses neither to save nor to borrow.

(a) Suppose the interest rate on bank loans goes up to 30% and the interest rate on bonds fall to 5%. Write down the equation of the new budget constraint and draw his budget line.
(b) Will he lend or borrow? By how much?

Test code: ME I/ME II, 2009

Syllabus for ME I, 2009

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2009

1. An infinite geometric series has first term 1 and sum 4. Its common ratio is

   A $\frac{1}{2}$

   B $\frac{3}{4}$

   C 1

   D $\frac{1}{3}$

2. A continuous random variable X has a probability density function $f(x) = 3x^2$ with $0 \leq x \leq 1$. If $P(X \leq a) = P(x > a)$, then a is:

   A $\frac{1}{\sqrt{6}}$

   B $\left(\frac{1}{3}\right)^{\frac{1}{2}}$

   C $\frac{1}{2}$

   D $\left(\frac{1}{2}\right)^{\frac{1}{3}}$

3. If $f(x) = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x + \ldots}}}$ then $f'(x)$ equals to

   A $\frac{f(x)-1}{2f(x)+1}$.

   B $\frac{f^2(x)-f(x)}{2f(x)-1}$.

   C $\frac{2f(x)+1}{f^2(x)+f(x)}$.

   D $\frac{f(x)}{2f(x)+1}$.

4. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$ is

   A $\frac{1}{6}$

   B 0

   C $\frac{1}{4}$

   D not well defined

5. If $X = 2^{65}$ and $Y = 2^{64} + 2^{63} + \ldots + 2^1 + 2^0$, then

   A $Y = X + 2^{64}$.

B $X = Y$.

C $Y = X + 1$.

D $Y = X - 1$.

6. $\int_0^1 \frac{e^x}{e^x+1} dx =$

A $log(1 + e)$.

B $log2$.

C $log\frac{1+e}{2}$.

D $2log(1 + e)$.

7. There is a box with ten balls. Each ball has a number between 1 and 10 written on it. No two balls have the same number. Two balls are drawn (simultaneously) at random from the box. What is the probability of choosing two balls with odd numbers?

A $\frac{1}{9}$.

B $\frac{1}{2}$.

C $\frac{2}{9}$.

D $\frac{1}{3}$.

8. A box contains 100 balls. Some of them are white and the remaining are red. Let X and Y denote the number of white and red balls respectively. The correlation between X and Y is

A 0.

B 1.

C -1.

D some real number between $-\frac{1}{2}$ and $\frac{1}{2}$.

9. Let f, g and h be real valued functions defined as follows: $f(x) = x(1 - x)$, $g(x) = \frac{x}{2}$ and $h(x) = \min\{f(x), g(x)\}$ with $0 \leq x \leq 1$. Then h is

A continuous and differentiable

B is differentiable but not continuous

C is continuous but not differentiable

D is neither continuous nor differentiable

10. In how many ways can three persons, each throwing a single die once, make a score of 8?

A 5

B 15

C 21

D 30

11. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 55 - 7x,$$

for every $x \in \Re$, then $f(3)$ equals

A 40

B 32

C 26

D 10

12. Two persons, A and B, make an appointment to meet at the train station between 4 P.M. and 5 P.M.. They agree that each is to wait not more than 15 minutes for the other. Assuming that each is independently equally likely to arrive at any point during the hour, find the probability that they meet.

A $\frac{15}{16}$

B $\frac{7}{16}$

C $\frac{5}{24}$

D $\frac{22}{175}$

13. If x_1, x_2, x_3 are positive real numbers, then

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1}$$

is always

A ≤ 3

B $\leq 3^{\frac{1}{3}}$

C ≥ 3

D 3

14. $\lim_{n\to\infty} \frac{1^2+2^2+\ldots+n^2}{n^3}$ equals

A 0

B $\frac{1}{3}$

C $\frac{1}{6}$

D 1.

15. Suppose b is an odd integer and the following two polynomial equations have a common root.

$$x^2 - 7x + 12 = 0$$
$$x^2 - 8x + b = 0.$$

The root of $x^2 - 8x + b = 0$ that is not a root of $x^2 - 7x + 12 = 0$ is

A 2

B 3

C 4

D 5

16. Suppose $n \geq 9$ is an integer. Let $\mu = n^{\frac{1}{2}} + n^{\frac{1}{3}} + n^{\frac{1}{4}}$. Then, which of the following relationships between n and μ is correct?

A $n = \mu$.

B $n > \mu$.

C $n < \mu$.

D None of the above.

17. Which of the following functions $f : \Re \to \Re$ satisfies the relation $f(x+y) = f(x) + f(y)$?

A $f(z) = z^2$

B $f(z) = az$ for some real number a

C $f(z) = \log z$

D $f(z) = e^z$

18. For what value of a does the following equation have a unique solution?

$$\begin{vmatrix} x & a & 2 \\ 2 & x & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

A 0

B 1

C 2

D 4

19. Let

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

where l, m, n, a, b, c are non-zero numbers. Then $\frac{dy}{dx}$ equals

A

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

B

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix}$$

C

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

D

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l-a & m-b & n-c \\ 1 & 1 & 1 \end{vmatrix}$$

20. If $f(x) = |x-1| + |x-2| + |x-3|$, then $f(x)$ is differentiable at

A 0

B 1

C 2

D 3

21. If $(x-a)^2+(y-b)^2=c^2$, then $1+\left[\frac{dy}{dx}\right]^2$ is independent of

A a

B b

C c

D Both b and c.

22. A student is browsing in a second-hand bookshop and finds n books of interest. The shop has m copies of each of these n books. Assuming he never wants duplicate copies of any book, and that he selects at least one book, how many ways can he make a selection? For example, if there is one book of interest with two copies, then he can make a selection in 2 ways.

A $(m+1)^n-1$

B nm

C $2^{nm}-1$

D $\frac{nm!}{(m!)(nm-m)!}-1$

23. Determine all values of the constants A and B such that the following function is continuous for all values of x.

$$f(x)=\begin{cases} Ax-B & \text{if } x\le -1 \\ 2x^2+3Ax+B & \text{if } -1<x\le 1 \\ 4 & \text{if } x>1 \end{cases}$$

A $A=B=0$

B $A=\frac{3}{4}$, $B=-\frac{1}{4}$

C $A=\frac{1}{4}$, $B=\frac{3}{4}$

D $A=\frac{1}{2}$, $B=\frac{1}{2}$

24. The value of $\lim_{x\to\infty}(3^x+3^{2x})^{\frac{1}{x}}$ is

A 0

B 1

C e

D 9

25. A computer while calculating correlation coefficient between two random variables X and Y from 25 pairs of observations obtained the following results: $\sum X = 125$, $\sum X^2 = 650$, $\sum Y = 100$, $\sum Y^2 = 460$, $\sum XY = 508$. It was later discovered that at the time of inputing, the pair $(X = 8, Y = 12)$ had been wrongly input as $(X = 6, Y = 14)$ and the pair $(X = 6, Y = 8)$ had been wrongly input as $(X = 8, Y = 6)$. Calculate the value of the correlation coefficient with the correct data.

A $\frac{4}{5}$

B $\frac{2}{3}$

C 1

D $\frac{5}{6}$

26. The point on the curve $y = x^2 - 1$ which is nearest to the point $(2, -0.5)$ is

A $(1, 0)$

B $(2, 3)$

C $(0, -1)$

D None of the above

27. If a probability density function of a random variable X is given by $f(x) = kx(2 - x), 0 \leq x \leq 2$, then mean of X is

A $\frac{1}{2}$

B 1

C $\frac{1}{5}$

D $\frac{3}{4}$

28. Suppose X is the set of all integers greater than or equal to 8. Let $f : X \rightarrow \Re$. and $f(x + y) = f(xy)$ for all $x, y \geq 4$. If $f(8) = 9$, then $f(9) =$

A 8.

B 9.

C 64.

D 81.

29. Let $f : \Re \rightarrow \Re$ be defined by $f(x) = (x-1)(x-2)(x-3)$. Which of the following is true about f?

A It decreases on the interval $[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}]$

B It increases on the interval $[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}]$

C It decreases on the interval $(-\infty, 2 - 3^{-\frac{1}{2}}]$

D It decreases on the interval $[2, 3]$

30. A box with no top is to be made from a rectangular sheet of cardboard measuring 8 metres by 5 metres by cutting squares of side x metres out of each corner and folding up the sides. The largest possible volume in cubic metres of such a box is

A 15

B 12

C 20

D 18

Syllabus for ME II (Economics), 2009

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2009

1. Consider the following model of the economy:

$$\begin{aligned} C &= c_0 + c_1 Y_D \\ T &= t_0 + t_1 Y \\ Y_D &= Y - T. \end{aligned}$$

C denotes consumption, $c_0 > 0$ denotes autonomous consumption, $0 < c_1 < 1$ is the marginal propensity to consume, Y, denotes income, T denotes taxes, Y_D denotes disposable income and $t_0 > 0$, $t_1 > 0$. Assume a closed economy where government spending G, and investment I, are exogenously given by $\bar{G}$ and $\bar{I}$ respectively.

- (i) Interpret t_1 in words. Is it greater or less than 1? Explain your answer.
- (ii) Solve for equilibrium output, Y^*.
- (iii) What is the multiplier ? Does the economy respond more to changes in autonomous spending (such as changes in $c_0, \bar{G}$, and, $\bar{I}$) when t_1 is zero or when t_1 is positive ? Explain.

[5]+[5]+[10]

2. Consider an agent who values consumption in periods 0 and 1 according to the utility function

$$u(c_0, c_1) = \log c_0 + \delta \log c_1$$

where $0 < \delta < 1$. Suppose that the agent has wealth ω in period 0 of which she can save any portion in order to consume in period 1. If she saves Re. 1, she is paid interest r so that her budget constraint is

$$c_0 + \tfrac{c_1}{1+r} = \omega$$

- (i) Derive the agent's demand for c_0 and c_1 as a function of r and ω.
- (ii) What happens to c_0 and c_1 as r increases? Interpret.
- (iii) For what relationship between ω and r will she consume the same amount in both periods?

  [8]+[6]+[6]

3. Consider a firm with production function $F(x_1, x_2) = \min(2x_1, x_1 + x_2)$ where x_1 and x_2 are amounts of factors 1 and 2.

- (i) Draw an isoquant for output level 10.

- (ii) Show that the production function exhibits constant returns to scale.
- (iii) Suppose that the firm faces input prices $w_1 = w_2 = 1$. What is the firms' cost function?

  [8]+[6]+[6]

4. Consider an exchange economy consisting of two individuals 1 and 2, and two goods X and Y. The utility function of individual i, $U_i = X_i + Y_i$. Individual 1 has 3 units of X and 7 units of Y to begin with. Similarly, individual 2 has 7 units of X and 3 units of Y to begin with.

- (i) What is the set of Pareto optimal outcomes in this economy? Justify your answer.
- (ii) What is the set of perfectly competitive (Walrasian) outcomes? You may use diagrams for parts (i) and (ii).
- (iii) Are the perfectly competitive outcomes Pareto optimal? Does this result hold generally in all exchange economies?

  [8]+[8]+[4]

5. A monopoly sells its product in two separate markets. The inverse demand function in market 1 is given by $q_1 = 10 - p_1$, and the inverse demand function in market 2 is given by $q_2 = a - p_2$, where $10 < a \leq 20$. The monopolist's cost function is $C(q) = 5q$, where q is aggregate output.

- (i) Suppose the monopolist must set the same price in both markets. What is its optimal price? What is the reason behind the restriction that $a \leq 20$?
- (ii) Suppose the monopolist can charge different prices in the two markets. Compute the prices it will set in the two markets.
- (iii) Under what conditions does the monopolist benefit from the ability to charge different prices?
- (iv) Compute consumers' surplus in cases (i) and (ii). Who benefits from differential pricing and who does not relative to the case where the same price is charged in both markets?

[5]+[5]+[5]+[5]

6. Consider an industry with 3 firms, each having marginal cost equal to 0. The inverse demand curve facing this industry is $p = 120 - q$, where q is aggregate output.

- (i) If each firm behaves as in the Cournot model, what is firm 1's optimal output choice as a function of its beliefs about other firms' output choices?
- (ii) What output do the firms produce in equilibrium?
- (iii) Firms 2 and 3 decide to merge and form a single firm with marginal cost still equal to 0. What output do the two firms produce in equilibrium? Is firm 1 better off as a result? Are firms 2 and 3 better off post-merger? Would it be better for all the firms to form a cartel instead? Explain in each case.

[3]+[5]+[12]

7. Suppose the economy's production function is given by

$$Y_t = 0.5\sqrt{K_t}\sqrt{N_t} \tag{1}$$

Y_t denotes output, K_t denotes the aggregate capital stock in the economy, and N denotes the number of workers (which is fixed). The evolution of the capital stock is given by,

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{2}$$

where the savings rate of the economy is denoted by, s, and the depreciation rate is given by, δ.

- (i) Using equation (2), show that the change in the capital stock per worker, $\frac{K_{t+1}-K_t}{N}$, is equal to savings per worker minus depreciation per worker.
- (ii) Derive the economy's steady state levels of $\frac{K}{N}$ and $\frac{Y}{N}$ in terms of the savings rate and the depreciation rate.
- (iii) Derive the equation for the steady state level of consumption per worker in terms of the savings rate and the depreciation rate.
- (iv) Is there a savings rate that is optimal, i.e., maximizes steady state consumption per worker ? If so, derive an expression for the optimal savings rate. Using words and graphs, discuss your answer.

[2]+[6]+[6]+[6]

8. Suppose there are 10 individuals in a society, 5 of whom are of high ability, and 5 of low ability. Individuals know their own abilities. Suppose that each individual lives for two periods and is deciding whether or not to go to college in period 1. When individuals make decisions in period 1, they choose that option which gives the highest lifetime payoff, i.e., the sum of earnings and expenses in both periods.

Education can only be acquired in period 1. In the absence of schooling, high and low ability individuals can earn y_H and y_L respectively in each period.

With education, period 2 earning increases to $(1 + a)y_H$ for high ability types and $(1 + a)y_L$ for low ability types. Earnings would equal 0 in period 1 if an individual decided to go to college in that period. Tuition fee for any individual is equal to T. Assume y_H and y_L are both positive, as is T.

- (i) Find the condition that determines whether each type of person will go to college in period 1. What is the minimum that a can be if it is to be feasible for any type of individual to acquire education?
- (ii) Suppose $y_H = 50$, $y_L = 40$, $a = 3$. For what values of T will a high ability person go to college? And a low ability person? Which type is more likely to acquire education?
- (iii) Now assume the government chooses to subsidise education by setting tuition equal to 60. What happens to educational attainment?
- (iv) Suppose now to pay for the education subsidy, the government decides to impose a $x\%$ tax on earnings in any period greater than 50. So if an individual earns 80 in a period, he would pay a tax in that period equal to $x\%$ of 30.The government wants all individuals to acquire education, and also wants to cover the cost of the education subsidy in period 1 through tax revenues collected in both periods. What value of x should the government set?

[5]+[5]+[2]+[8]

9. Consider the goods market with exogenous (constant) investment $\bar{I}$, exogenous government spending, $\bar{G}$ and constant taxes, T. The consumption equation is given by,

$$C = c_0 + c_1(Y - T),$$

where C denotes consumption, c_0 denotes autonomous consumption, and c_1 the marginal propensity to consume.

- (i) Solve for equilibrium output. What is the value of the multiplier ?
- (ii) Now let investment depend on Y and the interest rate, i

$$I = b_0 + b_1 Y - b_2 i,$$

  where b_0 and b_1 are parameters. Solve for equilibrium output. At a given interest rate, is the effect of an increase in autonomous spending bigger than it was in part (i)? In answering this, assume that $c_1 + b_1 < 1$.
- (iii) Now, introduce the financial market equilibrium condition

$$\frac{M}{P} = d_1 Y - d_2 i,$$

  where $\frac{M}{P}$ denotes the real money supply. Derive the multiplier. Assume that investment is given by the equation in part (ii).

- (iv) Is the multiplier you obtained in part (iii) smaller or larger than the multiplier you obtained in part (i). Explain how your answer depends on the behavioral equations for consumption, investment, and money demand.

[5]+[5]+[5]+[5]

10 (i) A college is trying to fill one remaining seat in its Masters programme. It judges the merit of any applicant by giving him an entrance test. It is known that there are two interested applicants who will apply sequentially. If the college admits the first applicant, it cannot admit the second. If it rejects the first applicant, it must admit the second. It is not possible to delay a decision on the first applicant till the second applicant is tested. At the time of admitting or rejecting the first applicant, the college thinks the second applicant's mark will be a continuous random variable drawn from the uniform distribution between 0 and 100. (Recall that a random variable x is uniformly distibuted on $[a, b]$ if the density function of x is given by $f(x) = \frac{1}{b-a}$ for $x \in [a, b]$). If the college wants to maximize the expected mark of its admitted student, what is the lowest mark for which it should admit the first applicant?

(ii) Now suppose there are three applicants who apply sequentially. Before an applicant is tested, it is known that his likely mark is an independent continuous random variable drawn from the uniform distribution between 0 and 100. What is the lowest mark for which the college should admit the first student? What is the lowest mark for which the college should admit the second student in case the first is rejected?
[8]+[12]

Test code: ME I/ME II, 2010

Syllabus for ME I, 2010

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2010

1. The value of $100\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{99.100}\right]$

   (a) is 99,

   (b) is 100,

   (c) is 101,

   (d) is $\frac{(100)^2}{99}$.

2. The function $f(x) = x(\sqrt{x}+\sqrt{x+9})$ is

   (a) continuously differentiable at $x = 0$,

   (b) continuous but not differentiable at $x = 0$,

   (c) differentiable but the derivative is not continuous at $x = 0$,

   (d) not differentiable at $x = 0$.

3. Consider a GP series whose first term is 1 and the common ratio is a positive integer $r(> 1)$. Consider an AP series whose first term is 1 and whose $(r+2)^{\text{th}}$ term coincides with the third term of the GP series. Then the common difference of the AP series is

   (a) $r-1$,

   (b) r,

   (c) $r+1$,

   (d) $r+2$.

4. The first three terms of the binomial expansion $(1+x)^n$ are $1, -9, \frac{297}{8}$ respectively. What is the value of n?

   (a) 5

   (b) 8

   (c) 10

   (d) 12

5. Given $log_p x = \alpha$ and $log_q x = \beta$, the value of $log_{\frac{p}{q}} x$ equals

   (a) $\frac{\alpha\beta}{\beta-\alpha}$,

   (b) $\frac{\beta-\alpha}{\alpha\beta}$,

   (c) $\frac{\alpha-\beta}{\alpha\beta}$,

   (d) $\frac{\alpha\beta}{\alpha-\beta}$.

6. Let $P = \{1, 2, 3, 4, 5\}$ and $Q = \{1, 2\}$. The total number of subsets X of P such that $X \cap Q = \{2\}$ is

   (a) 6,
   (b) 7,
   (c) 8,
   (d) 9.

7. An unbiased coin is tossed until a head appears. The expected number of tosses required is

   (a) 1,
   (b) 2,
   (c) 4,
   (d) ∞.

8. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x \geq c \\ 0 & x < c. \end{cases}$$

   Then the expectation of X is

   (a) 0,
   (b) ∞,
   (c) $\frac{1}{c}$,
   (d) $\frac{1}{c^2}$.

9. The number of real solutions of the equation $x^2 - 5|x| + 4 = 0$ is

   (a) two,
   (b) three,
   (c) four,
   (d) None of these.

10. Range of the function $f(x) = \frac{x^2}{1+x^2}$ is

   (a) $[0, 1)$,
   (b) $(0, 1)$,
   (c) $[0, 1]$.
   (d) $(0, 1]$.

11. If a, b, c are in AP, then the value of the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

is

(a) $b^2 - 4ac$,

(b) $ab + bc + ca$,

(c) $2b - a - c$,

(d) $3b + a + c$.

12. If $a < b < c < d$, then the equation $(x-a)(x-b) + 2(x-c)(x-d) = 0$ has

(a) both the roots in the interval $[a, b]$,

(b) both the roots in the interval $[c, d]$,

(c) one root in the interval (a, b) and the other root in the interval (c, d),

(d) one root in the interval $[a, b]$ and the other root in the interval $[c, d]$.

13. Let f and g be two differentiable functions on $(0, 1)$ such that $f(0) = 2$, $f(1) = 6$, $g(0) = 0$ and $g(1) = 2$. Then there exists $\theta \in (0, 1)$ such that $f'(\theta)$ equals

(a) $\frac{1}{2}g'(\theta)$,

(b) $2g'(\theta)$,

(c) $6g'(\theta)$,

(d) $\frac{1}{6}g'(\theta)$.

14. The minimum value of $log_x a + log_a x$, for $1 < a < x$, is

(a) less than 1,

(b) greater than 2,

(c) greater than 1 but less than 2.

(d) None of these.

15. The value of $\int_4^9 \frac{1}{2x(1+\sqrt{x})} dx$ equals

(a) $log_e 3 - log_e 2$,

(b) $2log_e 2 - log_e 3$,

(c) $2log_e 3 - 3log_e 2$,

(d) $3log_e 3 - 2log_e 2$.

16. The inverse of the function $f(x) = \frac{1}{1+x}$, $x > 0$, is

(a) $(1 + x)$,
(b) $\frac{1+x}{x}$,
(c) $\frac{1-x}{x}$,
(d) $\frac{x}{1+x}$.

17. Let X_i, $i = 1, 2, \ldots, n$ be identically distributed with variance σ^2. Let $cov(X_i, X_j) = \rho$ for all $i \neq j$. Define $\bar{X}_n = \frac{1}{n} \sum X_i$ and let $a_n = Var(\bar{X}_n)$. Then $\lim_{n\to\infty} a_n$ equals

(a) 0,
(b) ρ,
(c) $\sigma^2 + \rho$,
(d) $\sigma^2 + \rho^2$.

18. Let X be a Normally distributed random variable with mean 0 and variance 1. Let $\Phi(.)$ be the cumulative distribution function of the variable X. Then the expectation of $\Phi(X)$ is

(a) $-\frac{1}{2}$,
(b) 0,
(c) $\frac{1}{2}$,
(d) 1.

19. Consider any finite integer $r \geq 2$. Then $\lim_{x\to 0} \left[\frac{log_e \left(\sum_{k=0}^{r} x^k \right)}{\left(\sum_{k=1}^{\infty} \frac{x^k}{k!} \right)} \right]$ equals

(a) 0,
(b) 1,
(c) e,
(d) $log_e 2$.

20. Consider 5 boxes, each containing 6 balls labelled $1, 2, 3, 4, 5, 6$. Suppose one ball is drawn from each of the boxes. Denote by b_i, the label of the ball drawn from the i-th box, $i = 1, 2, 3, 4, 5$. Then the number of ways in which the balls can be chosen such that $b_1 < b_2 < b_3 < b_4 < b_5$ is

(a) 1,
(b) 2,
(c) 5,
(d) 6.

21. The sum $\sum_{r=0}^{m} \binom{n+r}{r}$ equals

(a) $\binom{n+m+1}{n+m}$,

(b) $(n+m+1)\binom{n+m}{n+1}$,

(c) $\binom{n+m+1}{n}$,

(d) $\binom{n+m+1}{n+1}$.

22. Consider the following 2-variable linear regression where the error ϵ_i's are independently and identically distributed with mean 0 and variance 1,

$$y_i = \alpha + \beta(x_i - \bar{x}) + \epsilon_i, \qquad i = 1, 2, \ldots, n.$$

Let $\hat{\alpha}$ and $\hat{\beta}$ be ordinary least squares estimates of α and β respectively. Then the correlation coefficient between $\hat{\alpha}$ and $\hat{\beta}$ is

(a) 1,

(b) 0,

(c) -1,

(d) $\frac{1}{2}$.

23. Let f be a real valued continuous function on $[0, 3]$. Suppose that $f(x)$ takes only rational values and $f(1) = 1$. Then $f(2)$ equals

(a) 2,

(b) 4,

(c) 8.

(d) None of these.

24. Consider the function $f(x_1, x_2) = \int_0^{\sqrt{x_1^2+x_2^2}} e^{-(w^2/(x_1^2+x_2^2))} dw$ with the property that $f(0, 0) = 0$. Then the function $f(x_1, x_2)$ is

(a) homogeneous of degree -1,

(b) homogeneous of degree $\frac{1}{2}$,

(c) homogeneous of degree 1.

(d) None of these.

25. If $f(1) = 0$, $f'(x) > f(x)$ for all $x > 1$, then $f(x)$ is

(a) positive valued for all $x > 1$,

(b) negative valued for all $x > 1$,

(c) positive valued on $(1, 2)$ but negative valued on $[2, \infty)$.

(d) None of these.

26. Consider the constrained optimization problem

$$\max_{x\geq 0, y\geq 0} (ax + by) \text{ subject to } (cx + dy) \leq 100$$

where a, b, c, d are positive real numbers such that $\frac{d}{b} > \frac{(c+d)}{(a+b)}$. The unique solution (x^*, y^*) to this constrained optimization problem is

(a) $(x^* = \frac{100}{a}, y^* = 0)$,

(b) $(x^* = \frac{100}{c}, y^* = 0)$,

(c) $(x^* = 0, y^* = \frac{100}{b})$,

(d) $(x^* = 0, y^* = \frac{100}{d})$.

27. For any real number x, let $[x]$ be the largest integer not exceeding x. The domain of definition of the function $f(x) = \left(\sqrt{[|[x| - 2]| - 3}\right)^{-1}$ is

(a) $[-6, 6]$,

(b) $(-\infty, -6) \cup (+6, \infty)$,

(c) $(-\infty, -6] \cup [+6, \infty)$,

(d) None of these.

28. Let $f : \Re \to \Re$ and $g : \Re \to \Re$ be defined as

$$f(x) = \begin{cases} -1 & \text{if } x < -\frac{1}{2} \\ -\frac{1}{2} & \text{if } -\frac{1}{2} \leq x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0. \end{cases}$$

and $g(x) = 1 + x - [x]$, where $[x]$ is the largest integer not exceeding x. Then $f(g(x))$ equals

(a) -1,

(b) $-\frac{1}{2}$,

(c) 0,

(d) 1.

29. If f is a real valued function and $a_1 f(x) + a_2 f(-x) = b_1 - b_2 x$ for all x with $a_1 \neq a_2$ and $b_2 \neq 0$. Then $f(\frac{b_1}{b_2})$ equals

(a) 0,

(b) $-\left(\frac{2a_2 b_1}{a_1^2 - a_2^2}\right)$,

(c) $\frac{2a_2 b_1}{a_1^2 - a_2^2}$.

(d) More information is required to find the exact value of $f(\frac{b_1}{b_2})$.

30. For all $x, y \in (0, \infty)$, a function $f : (0, \infty) \to \Re$ satisfies the inequality

$$|f(x) - f(y)| \leq |x - y|^3.$$

Then f is

(a) an increasing function,

(b) a decreasing function,

(c) a constant function.

(d) None of these.

Syllabus for ME II (Economics), 2010

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2010

1.

(a) Imagine a closed economy in which tax is imposed only on income. The government spending (G) is required (by a balanced budget amendment to the relevant law) to be equal to the tax revenue; thus $G = tY$, where t is the tax rate and Y is income. Consumption expenditure (C) is proportional to disposable income and investment (I) is exogenously given.

(i) Explain why government spending is endogenous in this model.

(ii) Is the multiplier in this model larger or smaller than in the case in which government spending is exogenous?

(iii) When t increases, does Y decrease, increase or stay the same? Give an answer with intuitive explanation.

(b) Consider the following macroeconomic model with notation having usual meanings: $C = 100 + 1.3Y$ (Consumption function), $I = \frac{500}{r}$ (Investment function), $M^D = 150Y + 100 - 1500r$ (Demand for money function) and $M^S = 2100$ (Supply of money). Do you think that there exists an equilibrium? Justify your answer using the IS-LM model.

[3+8+4]+[5]

2. Consider a market with two firms. Let the cost function of each firm be $C(q) = mq$ where $q \geq 0$. Let the inverse demand functions of firms 1 and 2 be $P_1(q_1, q_2) = a - q_1 - sq_2$ and $P_2(q_1, q_2) = a - q_2 - sq_1$, respectively. Assume that $0 < s < 1$ and $a > m > 0$.

(a) Find the Cournot equilibrium quantities of the two firms.

(b) Using the inverse demand functions $P_1(q_1, q_2)$ and $P_2(q_1, q_2)$, derive direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ of firms 1 and 2.

(c) Using the direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$, find the Bertrand equilibrium prices.

[8]+[5]+[7]

3.

(a) A monopolist can sell his output in two geographically separated markets A and B. The total cost function is $TC = 5 + 3(Q_A + Q_B)$ where Q_A and Q_B are quantities sold in markets A and B respectively. The demand functions for the two markets are, respectively, $P_A = 15 - Q_A$ and $P_B = 25 - 2Q_B$. Calculate the firm's price, output, profit and the deadweight loss to the society if it can get involved in price discrimination.

(b) Suppose that you have the following information. Each month an airline sells 1500 business-class tickets at Rs. 200 per ticket and 6000 economy-class tickets at Rs. 80 per ticket. The airline treats business class and economy class as two separate markets. The airline knows the demand curves for the two markets and maximizes profit. It is also known that the demand curve of each of the two markets is linear and marginal cost associated with each ticket is Rs. 50.

(i) Use the above information to construct the demand curves for economy class and business class tickets.

(ii) What would be the equilibrium quantities and prices if the airline could not get involved in price discrimination?

[12]+**[4+4]**

4. Consider an economy producing two goods 1 and 2 using the following production functions: $X_1 = L_1^{\frac{1}{2}} K^{\frac{1}{2}}$ and $X_2 = L_2^{\frac{1}{2}} T^{\frac{1}{2}}$, where X_1 and X_2 are the outputs of good 1 and 2, respectively, K is capital used in production of good 1, T is land used in production of good 2 and L_1 and L_2 are amounts of labour used in production of good 1 and 2, respectively. Full employment of all factors is assumed implying the following: $K = \bar{K}, T = \bar{T}, L_1 + L_2 = \bar{L}$ where $\bar{K}$, $\bar{T}$ and $\bar{L}$ are total amounts of capital, land and labour available to the economy. Labour is assumed to be perfectly mobile between sectors 1 and 2. The underlying preference pattern of the economy generates the relative demand function, $\frac{D_1}{D_2} = \gamma \left(\frac{p_1}{p_2}\right)^{-2}$, where D_1 and D_2 are the demands and p_1 and p_2 prices of good 1 and 2 respectively. All markets (both commodities and factors) are competitive.

(a) Derive the relationship between $\frac{X_1}{X_2}$ and $\frac{p_1}{p_2}$.

(b) Suppose that γ goes up. What can you say about the new equilibrium relative price?

[15]+**[5]**

5. Consider the IS-LM representation of an economy with the following features:

(i) The economy is engaged in export and import of goods and services, but not in capital transactions with foreign countries.
(ii) Nominal exchange rate, that is, domestic currency per unit of foreign currency, e, is flexible.
(iii) Foreign price level (P^*) and domestic price level (P) are given exogenously.
(iv) There is no capital mobility and e has to be adjusted to balance trade in equilibrium. The trade balance (TB) equation (with an autonomous part $\bar{T} > 0$) is given by $TB = \bar{T} + \frac{\beta P^*}{P} - mY$, where Y is GDP and β and m are positive parameters, m being the marginal propensity to import.

(a) Taking into account trade balance equilibrium and commodity market equilibrium, derive the relationship between Y and the interest rate (r). Is it the same as in the IS curve for the closed economy? Explain. Draw also the LM curve on the (Y, r) plane.

(b) Suppose that the government spending is increased. Determine graphically the new equilibrium value of Y. How does the equilibrium value of e change?

(c) Suppose that P^* is increased. How does it affect the equilibrium values of Y and e?

[9]+[6]+[5]

6.

(a) A firm can produce its product with two alternative technologies given by $Y = \min\{\frac{K}{3}, \frac{L}{2}\}$ and $Y = \min\{\frac{K}{2}, \frac{L}{3}\}$. The factor markets are competitive and the marginal cost of production is $Rs.20$ with each of these two technologies. Find the equation of the expansion path of the firm if it uses a third production technology given by $Y = K^{\frac{2}{3}} L^{\frac{1}{3}}$.

(b) A utility maximizing consumer with a given money income consumes two commodities X and Y. He is a price taker in the market for X. For Y there are two alternatives: (A) He purchases Y from the market being a price taker, (B) The government supplies a fixed quantity of it through ration shops free of cost. Is the consumer necessarily better off in case (B)? Explain your answer with respect to the following cases:

(i) Indifference curves are strictly convex to the origin.
(ii) X and Y are perfect substitutes.
(iii) X and Y are perfect complements.

[14]+[2+2+2]

7. Indicate, with adequate explanations, whether each of the following statements is TRUE or FALSE.

(a) If an increase in the price of a good leads a consumer to buy more of it, then an increase in his income will lead him to buy less of the good. By the same argument, if an increase in the price of the good leads him to buy less of it, then an increase in his income will lead him to buy more of the good.

(b) Suppose that a farmer, who receives all his income from the sale of his crop at a price beyond his control, consumes more of the crop as a result of the price increase. Then the crop is a normal good.

(c) If the non-wage income of a person increases then he chooses to work less at a given wage rate. Then he will choose to work more as his wage rate increases.

(d) The amount of stipends which Indian Statistical Institute pays to its students is a part of GDP.

[8]+[4]+[4]+[4]

8. Consider a Solow model with the production function $Y = K^{\frac{1}{2}}L^{\frac{1}{2}}$, where Y, K and L are levels of output, capital and labour, respectively. Suppose, 20% of income is saved and invested. Assume that the rate of growth of labour force, that is, $\left(\frac{\frac{dL}{dt}}{L}\right) = 0.05$.

(a) Find the capital-labour ratio, rate of growth of output, rate of growth of savings and the wage rate, in the steady state growth equilibrium.

(b) Suppose that the proportion of income saved goes up from 20% to 40%. What will be the new steady state growth rate of output?

(c) Is the rate of growth of output in the new steady state equilibrium different from that obtained just before attaining the new steady state (after deviating from the old steady state)? Explain.

[8]+[4]+[8]

9.

(a) Consider the utility function $U(x_1, x_2) = (x_1 - s_1)^{0.5}(x_2 - s_2)^{0.5}$, where $s_1 > 0$ and $s_2 > 0$ represent subsistence consumption and $x_1 \geq s_1$ and $x_2 \geq s_2$. Using the standard budget constraint, derive the budget share functions and demand functions of the utility maximizing consumer. Are they linear in prices? Justify your answer.

(b) Suppose that a consumer maximizes $U(x_1, x_2)$ subject to the budget constraint $p(x_1 + x_2) \leq M$ where $x_1 \geq 0$, $x_2 \geq 0$, $M > 0$ and $p > 0$. Moreover, assume that the utility function is symmetric, that is $U(x_1, x_2) = U(x_2, x_1)$ for all $x_1 \geq 0$ and $x_2 \geq 0$. If the solution (x_1^*, x_2^*) to the consumer's constrained optimization problem exists and is unique, then show that $x_1^* = x_2^*$.

[10]+[10]

10.

(a) Consider an economy with two persons (A and B) and two goods (1 and 2). Utility functions of the two persons are given by $U_A(x_{A1}, x_{A2}) = x_{A1}^{\alpha} + x_{A2}^{\alpha}$ with $0 < \alpha < 1$; and $U_B(x_{B1}, x_{B2}) = x_{B1} + x_{B2}$. Derive the equation of the contract curve and mention its properties.

(b) (i) A firm can produce a product at a constant average (marginal) cost of Rs. 4. The demand for the good is given by $x = 100 - 10p$. Assume that the firm owner requires a profit of Rs. 80. Determine the level of output and the price that yields maximum revenue if this profit constraint is to be fulfilled.

(ii) What will be the effects on price and output if the targeted profit is increased to Rs. 100?

(iii) Also find out the effects of the increase in marginal cost from Rs. 4 to Rs. 8 on price and output.

[8]+[6+3+3]

SYLLABUS & SAMPLE QUESTIONS FOR MS (QE)

2011

Syllabus for ME I, 2011

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2011

1. The expression $\sqrt{13+3\sqrt{23/3}}+\sqrt{13-3\sqrt{23/3}}$ is

   (a) A natural number,
   (b) A rational number but not a natural number,
   (c) An irrational number not exceeding 6,
   (d) An irrational number exceeding 6.

2. The domain of definition of the function $f(x)=\dfrac{\sqrt{(x+3)}}{(x^2+5x+4)}$ is

   (a) $(-\infty,\infty)\setminus\{-1,-4\}$,
   (b) $(-0,\infty)\setminus\{-1,-4\}$
   (c) $(-1,\infty)\setminus\{-4\}$
   (d) None of these.

3. The value of

$\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots\ldots.$

(a) $\log_e 2$, (b) $1 - \log_e 2$,
(c) $\log_e 2 - 1$, (d) None of these.

4. The function $\max\{1, x, x^2\}$, where x is any real number, has

(a) Discontinuity at one point only,
(b) Discontinuity at two points only,
(c) Discontinuity at three points only,
(d) No point of discontinuity.

5. If $x, y, z > 0$ are in HP, then $\dfrac{x-y}{y-z}$ equals

(a) $\dfrac{x}{y}$, (b) $\dfrac{y}{z}$, (c) $\dfrac{x}{z}$, (d) None of these.

6. The function $f(x) = \dfrac{x}{1+|x|}$, where x is any real number is,

(a) Everywhere differentiable but the derivative has a point of discontinuity.
(b) Everywhere differentiable except at 0.
(c) Everywhere continuously differentiable.
(d) Everywhere differentiable but the derivative has 2 points of discontinuity.

7. Let the function $f : R_{++} \to R_{++}$ be such that $f(1) = 3$ and $f'(1) = 9$, where R_{++} is the positive part of the real line. Then

$\lim_{x \to 0} \left(\dfrac{f(1+x)}{f(1)} \right)^{1/x}$ equals

(a) 3, (b) e^2, (c) 2, (d) e^3.

8. Let $f, g : [0, \infty) \to [0, \infty)$ be decreasing and increasing respectively. Define $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

(a) Nonpositive for $x \geq 1$, positive otherwise, (b) Always negative,
(c) Always positive, (d) Positive for $x \geq 1$, nonpositive otherwise.

9. A committee consisting of 3 men and 2 women is to be formed out of 6 men and 4 women. In how many ways this can be done if Mr. X and Mrs. Y are not to be included together?

(a) 120, (b) 140, (c) 90, (d) 60.

10. The number of continuous functions f satisfying $x\,f(y)+y\,f(x)=(x+y)\,f(x)\quad f(y)$, where x and y are any real numbers, is

(a) 1, (b) 2, (c), 3,
(d) None of these.

11. If the positive numbers $x_1, \ldots, x_n$ are in AP, then

$$\frac{1}{\sqrt{x_1}+\sqrt{x_2}}+\frac{1}{\sqrt{x_2}+\sqrt{x_3}}+\ldots\ldots+\frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}} \text{ equals}$$

(a) $\dfrac{n}{\sqrt{x_1}+\sqrt{x_n}}$, (b) $\dfrac{1}{\sqrt{x_1}+\sqrt{x_n}}$,

(b) $\dfrac{2n}{\sqrt{x_1}+\sqrt{x_n}}$, (d) None of these.

12. If x, y, z are any real numbers, then which of the following is always true?

(a) $\max\{x, y\} < \max\{x, y, z\}$,
(b) $\max\{x, y\} > \max\{x, y, z\}$,

(c) $\max\{x, y\} = \dfrac{x+y+|x-y|}{2}$

(d) None of these.

13. If $x_1, x_2, x_3, x_4 > 0$ and $\sum_{i=1}^{4} x_i = 2$, then $P = (x_1 + x_2)(x_3 + x_4)$ is

(a) Bounded between zero and one,
(b) Bounded between one and two,
(c) Bounded between two and three,
(d) Bounded between three and four.

14. Everybody in a room shakes hand with everybody else. Total number of handshakes is 91. Then the number of persons in the room is

(a) 11, (b) 12, (c) 13,
(d) 14.

15. The number of ways in which 6 pencils can be distributed between two boys such that each boy gets at least one pencil is

(a) 30, (b) 60, (c) 62, (d) 64.

16. Number of continuous functions characterized by the equation $x f(x) + 2 f(-x) = -1$, where x is any real number, is

(a) 1, (b) 2, (c) 3, (d) None of these.

17. The value of the function $f(x) = x + \int_0^1 (xy^2 + x^2 y) f(y) dy$ is

$px + qx^2$, where
(a) $p = 80$, $q = 180$,
(b) $p = 40$, $q = 140$
(c) p = 50, q = 150,
(d) None of these.

18. If x and y are real numbers such that $x^2 + y^2 = 1$, then the maximum value of $|x| + |y|$ is

(a) $\frac{1}{2}$, (b) $\sqrt{2}$, (c) $\frac{1}{\sqrt{2}}$, (d) 2.

19. The number of onto functions from $A = \{p, q, r, s\}$ to $B = \{p, r\}$ is

(a) 16, (b) 2, (c) 8, (d) 14.

20. If the coefficients of $(2r+5)$ th and $(r-6)$ th terms in the expansion of $(1+x)^{39}$ are equal, then ${}^{r}C_{12}$ equals

(a) 45, (b) 91, (c) 63, (d) None of these.

21. If $X = \begin{bmatrix} C & 2 \\ 1 & C \end{bmatrix}$ and $|X^7| = 128$, then the value of C is

(a) ± 5, (b) ± 1, (c) ± 2, (d) None of these.

22. Let $f(x) = Ax^2 + Bx + C$, where A, B, C re real numbers. If $f(x)$ is an integer whenever x is an integer, then

(a) 2 A and $A + B$ are integers, but C is not an integer.
(b) $A + B$ and C are integers, but 2 A is not an integer.
(c) 2 A, $A + B$ and C are all integers.
(d) None of these.

23. Four persons board a lift on the ground floor of a seven-storey building. The number of ways in which they leave the lift, such that each of them gets down at different floors, is

(a) 360, (b) 60, (c) 120, (d) 240.

24. The number of vectors (x, x_1, x_2), where $x, x_1, x_2 > 0$, for which

$$|\log(x x_1)| + |\log(x x_2)| + \left|\log\left(\frac{x}{x_1}\right)\right| + \left|\log\left(\frac{x}{x_2}\right)\right|$$

$= |\log x_1 + \log x_2|$ holds, is

(a) One, (b) Two, (c) Three, (d) None of these.

25. In a sample of households actually invaded by small pox, 70% of the inhabitants are attacked and 85% had been vaccinated. The minimum percentage of households (among those vaccinated) that

must have been attacked [Numbers expressed as nearest integer value] is

(a) 55, (b) 65, (c) 30, (d) 15.

26. In an analysis of bivariate data (X and Y) the following results were obtained.

Variance of $X\,(\sigma_x^2) = 9$, product of the regression coefficient of Y on X and X on Y is 0.36, and the regression coefficient from the regression of Y on X (β_{yx}) is 0.8.

The variance of Y is

(a) 16, (b) 4, (c) 1.69 ,d) 3.

27. For comparing the wear and tear quality of two brands of automobile tyres, two samples of 50 customers using two types of tyres under similar conditions were selected. The number of kilometers x_1 and x_2 until the tyres became worn out, was obtained from each of them for the tyres used by them. The sample results were as follows : $\bar{x}_1 = 13{,}200$ km, $\bar{x}_2 = 13{,}650$ km, S_{x1}= 300 km, S_{x2} = 400 km. What would you conclude about the two brands of tyres (at 5% level of significance) as far as the wear and tear quality is concerned?

(a) The two brands are alike ,
(b) The two brands are not the same,
(c) Nothing can be concluded,
(d) The given data are inadequate to perform a test.

28. A continuous random variable x has the following probability density function:

$$f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1} \quad \text{for } x > x_0, \ \alpha > 1.$$

The distribution function and the mean of x are given respectively by

(a) $1-\left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha-1}{\alpha}x_0$,

(b) $1-\left(\frac{x}{x_0}\right)^{-\alpha}$, $\frac{\alpha-1}{\alpha}x_0$,

(c) $1-\left(\frac{x}{x_0}\right)^{-\alpha}$, $\frac{\alpha x_0}{\alpha-1}$,

(d) $1-\left(\frac{x}{x_0}\right)^{\alpha}$, $\frac{\alpha x_0}{\alpha-1}$

29. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}$, $\binom{n}{1}$, $\ldots$, $\binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu=\frac{n}{2}$ and $\sigma^2=\frac{n}{2}$;

(B) $\mu=\frac{n}{4}$ and $\sigma^2=\frac{n}{4}$;

(C) $\mu=\frac{n}{2}$ and $\sigma^2=\frac{n}{4}$;

(D) $\mu=\frac{n}{4}$ and $\sigma^2=\frac{n}{2}$.

30. Let $\{X_i\}$ be a sequence of *i.i.d* random variables such that

$$X_i = 1 \text{ with probability } p$$
$$= 0 \text{ with probability } 1-p$$

Define $y=\begin{cases}1 & \text{if } \sum_{i=1}^{n} X_i = 100\\ 0 & \text{otherwise}\end{cases}$

Then $E(y^2)$ is

(a) ∞, (b) $\binom{n}{100}p^{100}(1-p)^{n-100}$, (c) np, (d) $(np)^2$.

Syllabus for ME II (Economics), 2011

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2011

1. A monopolist sells two products, X and Y. There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing); (ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

[30]

2. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan A.

   (a) Find equilibrium consumption under plan B.

   (b) Explain your answer if the equilibrium outcome in case (b) differs from that in case (a).

[6+18+6=30]

3. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ where C_t represents consumption or use of the resource at period t and δ ($0 < \delta < 1$) is the discount factor.

   (a) Set up the utility maximization problem of the community and interpret the first order condition.

   (b) Express the optimal consumption C_t for any period t in terms of the parameters δ and X.

   (c) If an unanticipated discovery of an additional stock of X' occurs at the beginning of period T ($0 < T < \infty$), what will be the new level of consumption at each period from T onwards?

[7+16+7=30]

4. A consumer, with a given money income M, consumes n goods $x_1, x_2, ..., x_n$ with given prices $p_1, p_2, ..., p_n$.

   (a) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Max(x_1, x_2, ..., x_n)$. Find the Marshallian demand function for good x_i and draw it in a graph.

   (b) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Min(x_1, x_2, ..., x_n)$. Find the income and the own price elasticities of demand for good x_i.

[15+15=30]

5. An economy, consisting of *m* individuals, is endowed with quantities $\omega_1, \omega_2, ...\omega_n$ of *n* goods. The *i*th individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good *j* of individual *i*.

   (a) Define an *allocation,* a *Pareto inferior allocation* and a *Pareto optimal allocation* for this economy.

   (b) Find an allocation which is *Pareto inferior* and an allocation which is *Pareto optimal.*

   (c) Consider an allocation where $C_j^i = \lambda^i \omega_j \forall j$, $\sum_i \lambda^i = 1$. Is this allocation *Pareto optimal*?

[6+18+6=30]

6. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - \frac{3}{2} q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Now, answer the following questions.

   (a) How much will the monopolist sell in the domestic market and how much will it sell in the foreign market?

   (b) Suppose, the home government imposes a restriction on the amount that the monopolist can sell in the foreign market. In particular, the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

[6+24=30]

7. An economy produces two goods, food (F) and manufacturing (M).

Food is produced by the production function $F = (L_F)^{\frac{1}{2}}(T)^{\frac{1}{2}}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{\frac{1}{2}}(K)^{\frac{1}{2}}$, where L_M is the labour employed, K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36$, $T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find out the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b) Next, we introduce a small change in the description of the economy given above. Assume, everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

(c) Suggest a measure of welfare for the economy as a whole.

(d) Using the above given data and your measure of welfare, determine whether the scenario given in problem (b), where land is owned by none, better or worse for the economy as a whole, compared to the scenario given in problem (a), where land is owned by the landlords?

(e) What do you think is the source of the difference in welfare levels (if any) under case (a) and case (b).

[6+10+4+6+4=30]

8. An economy produces a single homogeneous good in a perfectly competitive set up, using the production function $Y = AF(L, K)$, where Y is the output of the good, L and K are the amount of labour and capital respectively and A is the technological productivity parameter. Further, assume that F is homogeneous of degree one in L and K. Labour and capital in this economy remains fully employed. It has also been observed that the total wage earning of this economy is equal to the total earnings of capital in the economy at all points in time.

Answer the following questions.

(a) It is observed that over a given period the labour force grew by 4%, the capital stock grew by 3%, and output grew by 9%. What then was the growth rate of the technological productivity parameter (A) over that period?

(b) Over another period the wage rate of labour in this economy exhibited a growth of 30% , rental rate of capital grew by 10% and the price of the good over the same period grew by 5%. Find out the growth rate of the technological productivity parameter (A) over this period.

(c) Over yet another period, it was observed that there was no growth in the technological productivity, and the wages grew by 30% and rental rate grew by 10%. Infer from this, the growth rate of the price of the good over the period.

[4+20+6=30]

9. An economy produces two goods - *m* and *g*. Capitalists earn a total income, *R* (R_m from sector *m* plus R_g from sector *g*), but consumes only good *m*, spending a fixed proportion (*c*) of their income on it. Workers do *not* save. Workers in sector *m* spend a fixed proportion α of their income (*W*) on good *g* and the rest on good *m*. [However, whatever wages are paid in sector *g* are spent entirely for the consumption of good *g* only so that we ignore wages in this sector for computing both income generation therein and the expenditure made on its output.] The categories of income and expenditure in the two sectors are shown in detail in the chart below.

Sector *m* Sector *g*

Income generated	Expenditure on good *m*	Income (net of wages) generated	Expenditure (net of that by own workers) on good g
• Capitalists' income (R_m) • Wages (W)	• Capitalists' consumption (C = c. R) • Consumption of workers of sector *m*: ({1- α }.W) • Investment: (I)	Capitalists' Income (R_g)	• Consumption of workers of sector *m* (α .W)

Further, *investment* expenditure (I), made exclusively on *m*-good, is *autonomous* and income distribution in sector *m* is exogenously given:

$R_m = \theta . W$ (θ given).

(a) Equating aggregate income with aggregate expenditure for the economy, show that capitalists' income (R) is determined exclusively by their *own* expenditure (C and I). Is there any multiplier effect of I on R ? Give arguments.

(b) Show that I (along with c, α and θ) also determines$W$.

[15 +15= 30]

10. Consider two countries – a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per

unit and is in equilibrium initially (i.e. in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year *t* shows up as income in year *t* (Y_t), but consumption expenditure in year *t* (C_t) is given by: $C_t = 0.5\ Y_{t-1}$. The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at *zero* interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find out income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium in each of the following two alternative cases:

(a) The country takes the loan in year 1 only.

(b) The country takes the loan in year 1 and renews it every year.

[15 + 15=30]

SYLLABUS AND SAMPLE QUESTIONS FOR MS(QE)

2012

Syllabus for ME I (Mathematics), 2012

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions–Binomial and Normal.

Sample Questions for MEI (Mathematics), 2012

1. Kupamonduk, the frog, lives in a well 14 feet deep. One fine morning she has an urge to see the world, and starts to climb out of her well. Every day she climbs up by 5 feet when there is light, but slides back by 3 feet in the dark. How many days will she take to climb out of the well?
   (A) 3,
   (B) 8,
   (C) 6,
   (D) None of the above.

2. The derivative of $f(x) = |x|^2$ at $x = 0$ is,
   (A) -1,
   (B) Non-existent,
   (C) 0,
   (D) 1/2.

3. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathcal{N}$, define $A_n = \{(n+1)k : k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals
(A) A_3,
(B) A_4,
(C) A_5,
(D) A_6.

4. Let $S = \{a, b, c\}$ be a set such that a, b and c are distinct real numbers. Then $\min\{\max\{a, b\}, \max\{b, c\}, \max\{c, a\}\}$ is always
(A) the highest number in S,
(B) the second highest number in S,
(C) the lowest number in S,
(D) the arithmetic mean of the three numbers in S.

5. The sequence $< -4^{-n} >$, $n = 1, 2, \cdots$, is
(A) Unbounded and monotone increasing,
(B) Unbounded and monotone decreasing,
(C) Bounded and convergent,
(D) Bounded but not convergent.

6. $\int \frac{x}{7x^2+2} dx$ equals

(A) $\frac{1}{14} ln(7x^2 + 2)+$ constant,
(B) $7x^2 + 2$,
(C) $ln\ x+$ constant,
(D) None of the above.

7. The number of real roots of the equation

$$2(x-1)^2 = (x-3)^2 + (x+1)^2 - 8$$

is
(A) Zero,
(B) One,
(C) Two,
(D) None of the above.

8. The three vectors $[0,1]$, $[1,0]$ and $[1000,1000]$ are
(A) Dependent,
(B) Independent,
(C) Pairwise orthogonal,
(D) None of the above.

9. The function $f(.)$ is increasing over $[a,b]$. Then $[f(.)]^n$, where n is an odd integer greater than 1, is necessarily
(A) Increasing over $[a,b]$,
(B) Decreasing over $[a,b]$,
(C) Increasing over $[a,b]$ if and only if $f(.)$ is positive over $[a,b]$,
(D) None of the above.

10. The determinant of the matrix $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is
(A) 21,
(B) -16,
(C) 0,
(D) 14.

11. In what ratio should a given line be divided into two parts, so that the area of the rectangle formed by the two parts as the sides is the maximum possible?
(A) 1 is to 1,
(B) 1 is to 4,
(C) 3 is to 2,
(D) None of the above.

12. Suppose (x^*, y^*) solves:

$$Minimize \;\; ax + by,$$

subject to

$$x^\alpha + y^\alpha = M,$$

and $x, y \geq 0$, where $a > b > 0$, $M > 0$ and $\alpha > 1$. Then, the solution is,

(A) $\frac{x*^{\alpha-1}}{y*^{\alpha-1}} = \frac{a}{b}$,
(B) $x^* = 0,\ y^* = M^{\frac{1}{\alpha}}$,
(C) $y^* = 0,\ x^* = M^{\frac{1}{\alpha}}$,
(D) None of the above.

13. Three boys and two girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is
(A) $4_{P_2} \times 3!$,
(B) $3_{P_2} \times 2!$
(C) $2! \times 3!$
(D) None of the above.

14. The domain of x for which $\sqrt{x} + \sqrt{3-x} + \sqrt{x^2-4x}$ is real is,
(A) [0,3],
(B) (0,3),
(C) $\{0\}$,
(D) None of the above.

15. P (x) is a quadratic polynomial such that P (1) = P (-1). Then
(A) The two roots sum to zero,
(B) The two roots sum to 1,
(C) One root is twice the other,
(D) None of the above.

16. The expression $\sqrt{11+6\sqrt{2}} + \sqrt{11-6\sqrt{2}}$ is
(A) Positive and an even integer,
(B) Positive and an odd integer,
(C) Positive and irrational,
(D) None of the above.

17. What is the maximum value of $a(1-a)b(1-b)c(1-c)$, where a, b, c vary over all positive fractional values?

A 1,

B $\frac{1}{8}$,

C $\frac{1}{27}$,

D $\frac{1}{64}$.

18. There are four modes of transportation in Delhi: (A) Auto-rickshaw, (B) Bus, (C) Car, and (D) Delhi-Metro. The probability of using transports A, B, C, D by an individual is $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$, $\frac{2}{9}$ respectively. The probability that he arrives late at work if he uses transportation A, B, C, D is $\frac{5}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, and $\frac{6}{7}$ respectively. What is the probability that he used transport A if he reached office on time?

A $\frac{1}{9}$,

B $\frac{1}{7}$,

C $\frac{3}{7}$,

D $\frac{2}{9}$.

19. What is the least (strictly) positive value of the expression $a^3+b^3+c^3-3abc$, where a, b, c vary over all strictly positive integers? (You may use the identity $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\big((a-b)^2+(b-c)^2+(c-a)^2\big)$.)

A 2,

B 3,

C 4,

D 8.

20. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ is,
(A) -0.75,
(B) Belongs to the interval $[-1,\ -0.5]$,
(C) Belongs to the interval $[0.5,\ 1]$,
(D) None of the above.

21. Consider the following linear programming problem:
Maximize $a+b$ subject to
$a+2b \leq 4$,
$a+6b \leq 6$,

$a - 2b \leq 2$,
$a, b \geq 0$.
An optimal solution is:
(A) a=4, b=0,
(B) a=0, b=1,
(C) a=3,b=1/2,
(D) None of the above.

22. The value of $\int_{-4}^{-1} \frac{1}{x} dx$ equals,
(A) $ln\ 4$,
(B) Undefined,
(C) $ln(-4) - ln(-1)$,
(D) None of the above.

23. Given $x \geq y \geq z$, and $x + y + z = 9$, the maximum value of x + 3y + 5z is
(A) 27,
(B) 42,
(C) 21,
(D) 18.

24. A car with six sparkplugs is known to have two malfunctioning ones. If two plugs are pulled out at random, what is the probability of getting at least one malfunctioning plug.
(A) 1/15,
(B) 7/15,
(C) 8/15,
(D) 9/15.

25. Suppose there is a multiple choice test which has 20 questions. Each question has two possible responses - true or false. Moreover, only one of them is correct. Suppose a student answers each of them randomly.Which one of the following statements is correct?
(A) The probability of getting 15 correct answers is less than the probability of getting 5 correct answers,
(B) The probability of getting 15 correct answers is more than the

probability of getting 5 correct answers,
(C) The probability of getting 15 correct answers is equal to the probability of getting 5 correct answers,
(D) The answer depends on such things as the order of the questions.

26. From a group of 6 men and 5 women, how many different committees consisting of three men and two women can be formed when it is known that 2 of the men do not want to be on the committee together?
(A) 160,
(B) 80,
(C) 120,
(D) 200.

27. Consider any two consecutive integers a and b that are both greater than 1. The sum $(a^2 + b^2)$ is
(A) Always even,
(B) Always a prime number,
(C) Never a prime number,
(D) None of the above statements is correct.

28. The number of real non-negative roots of the equation

$$x^2 - 3|x| - 10 = 0$$

is,
(A) 2,
(B) 1,
(C) 0,
(D) 3.

29. Let $< a^n >$ and $< b^n >$, $n = 1, 2, \cdots$, be two different sequences, where $< a^n >$ is convergent and $< b^n >$ is divergent. Then the sequence $< a^n + b^n >$ is,
(A) Convergent,
(B) Divergent,
(C) Undefined,
(D) None of the above.

30. Consider the function

$$f(x) = \frac{|x|}{1+|x|}.$$

This function is,
(A) Increasing in x when $x \geq 0$,
(B) Decreasing in x,
(C) Increasing in x for all real x,
(D) None of the above.

Syllabus for ME II (Economics), 2012

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for ME II (Economics), 2012

1. A price taking firm makes machine tools Y using labour and capital according to the following production function

$$Y = L^{0.25}K^{0.25}.$$

Labour can be hired at the beginning of every week, while capital can be hired only at the beginning of every month. It is given that the wage rate = rental rate of capital = 10. Show that the short run (week) cost function is $10Y^4/K^*$ where the amount of capital is fixed at K^* and the long run (month) cost function is $20Y^2$.

2. Consider the following IS-LM model
$C = 200 + 0.25Y_D$,
$I = 150 + 0.25Y - 1000i$,

$G = 250,$
$T = 200,$
$(m/p)^d = 2Y - 8000i,$
$(m/p) = 1600,$

where C = aggregate consumption, I = investment, G = government expenditures, T = taxes, $(m/p)^d$ = money demand, (m/p) = money supply, Y_D = disposable income $(Y - T)$. Solve for the equilibrium values of all variables. How is the solution altered when money supply is increased to $(m/p) = 1840$? Explain intuitively the effect of expansionary monetary policy on investment in the short run.

3. Suppose that a price-taking consumer A maximizes the utility function $U(x, y) = x^\alpha + y^\alpha$ with $\alpha > 0$ subject to a budget constraint. Assume prices of both goods, x and y, are equal. Derive the demand function for both goods. What would your answer be if the price of x is twice that of the price of y?

4. Assume the production function for the economy is given by

$$Y = L^{0.5} K^{0.5}$$

where Y denotes output, K denotes the capital stock and L denotes labour. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ lies between 0 and 1 and is the rate of depreciation of capital. I represents investment, given by $I_t = sY_t$, where s is the savings rate. Derive the expression of steady state consumption and find out the savings rate that maximizes steady state consumption.

5. There are two goods x and y. Individual A has endowments of 25 units of good x and 15 units of good y. Individual B has endowments of 15 units of good x and 30 units of good y. The price of good y is Re. 1, no matter whether the individual buys or sells the good. The price of good x is Re. 1 if the individual wishes to sell it. It is, however, Rs. 3 if

the individual wishes to buy it. Let C_x and C_y denote the consumption of these goods. Suppose that individual B chooses to consume 20 units of good x and individual A does not buy or sell any of the goods and chooses to consume her endowment. Could A and B have the same preferences?

6. A monopolist has cost function $c(y) = y$ so that its marginal cost is constant at Re. 1 per unit. It faces the following demand curve

$$D(p) = \begin{cases} 0, \text{ if} & p > 20 \\ \dfrac{100}{p}, \text{ if} & p \leq 20. \end{cases}$$

Find the profit maximizing level of output if the government imposes a per unit tax of Re. 1 per unit, and also the dead-weight loss from the tax.

7. A library has to be located on the interval $[0, 1]$. There are three consumers A, B and C located on the interval at locations 0.3, 0.4 and 0.6, respectively. If the library is located at x, then A, B and C's utilities are given by $-|x-0.3|$, $-|x-0.4|$ and $-|x-0.6|$, respectively. Define a Pareto-optimal location and examine whether the locations $x = 0.1, x = 0.3$ and $x = 0.6$ are Pareto-optimal or not.

8. Consider an economy where the agents live for only two periods and where there is only one good. The life-time utility of an agent is given by $U = u(c) + \beta v(d)$, where u and v are the first and second period utilities, c and d are the first and second period consumptions and β is the discount factor. β lies between 0 and 1. Assume that both u and v are strictly increasing and concave functions. In the first period, income is w and in the second period, income is zero. The interest rate on savings carried from period 1 to period 2 is r. There is a government that taxes first period income. A proportion τ of income is taken away by the government as taxes. This is then returned in the second period to the agent as a lump sum transfer T. The government's budget is balanced i.e., $T = \tau w$. Set up the agent's optimization problem and write the first order condition assuming an interior solution. For given

values of r, β, w, show that increasing T will reduce consumer utility if the interest rate is strictly positive.

9. A monopolist sells two products, X and Y . There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4.

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing);(ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

10. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan B.

11. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the com-

munity maximizes an intertemporal utility function $U = \sum \delta^t ln(C_t)$ where C_t represents consumption or use of the resource at period t and $\delta(0 < \delta < 1)$ is the discount factor. Express the optimal consumption C_t for any period t in terms of the parameter δ and X.

12. A consumer, with a given money income M, consumes 2 goods x_1 and x_2 with given prices p_1 and p_2. Suppose that his utility function is $U(x_1, x_2) = Max(x_1, x_2)$. Find the Marshallian demand function for goods x_1, x_2 and draw it in a graph. Further, suppose that his utility function is $U(x_1, x_2) = Min(x_1, x_2)$. Find the income and the own price elasticities of demand for goods x_1 and x_2.

13. An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, ..., \omega_n$ of n goods. The ith individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good j of individual i.
(a) Define an allocation,a Pareto inferior allocation and a Pareto optimal allocation for this economy.
(b) Consider an allocation where $C_j^i = \lambda^i \omega_j$ for all j ,$\sum_i \lambda^i = 1$. Is this allocation Pareto optimal?

14. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - (\frac{3}{2})q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Suppose, that the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

15. An economy produces two goods, food (F) and manufacturing (M). Food is produced by the production function $F = (L_F)^{1/2}(T)^{1/2}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{1/2}(K)^{1/2}$, where L_M is the labour employed,

K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36, T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.
(a) Find the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)
(b)Next, we introduce a small change in the description of the economy given above. Assume that everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

16. Consider two countries - a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per unit and is in equilibrium initially (i.e., in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (say, Y_t) , but consumption expenditure in year t (say, C_t) is given by: $C_t = 0.5Y_{t-1}$.

    The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at zero interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find the income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium when the country takes the loan in year 1 only.

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)

2013

Syllabus for PEA (Mathematics), 2013

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample Questions for PEA (Mathematics), 2013

1. Let $f(x) = \frac{1-x}{1+x}, x \neq -1$. Then $f(f(\frac{1}{x})), x \neq 0$ and $x \neq -1$, is
(A) 1,
(B) x,
(C) x^2,
(D) $\frac{1}{x}$.

2. The limiting value of $\frac{1.2+2.3+\ldots+n(n+1)}{n^3}$ as $n \to \infty$ is,
(A) 0,
(B) 1,
(C) 1/3,
(D) 1/2.

3. Suppose $a_1, a_2, \ldots, a_n$ are n positive real numbers with $a_1 a_2 \ldots a_n = 1$. Then the minimum value of $(1 + a_1)(1 + a_2)\ldots(1 + a_n)$ is
(A) 2^n,
(B) 2^{2n},
(C) 1,
(D)None of the above.

4. Let the random variable X follow a Binomial distribution with parameters n and p where $n(> 1)$ is an integer and $0 < p < 1$. Suppose further that the probability of $X = 0$ is the same as the probability of $X = 1$. Then the value of p is
(A) $\frac{1}{n}$,
(B) $\frac{1}{n+1}$,
(C) $\frac{n}{n+1}$,
(D) $\frac{n-1}{n+1}$.

5. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100})$ is
(A) 1,
(B) 2^{100},

(C) 0,

(D) None of the above.

6. If α and β are the roots of the equation $x^2 - ax + b = 0$, then the quadratic equation whose roots are $\alpha + \beta + \alpha\beta$ and $\alpha\beta - \alpha - \beta$ is

(A) $x^2 - 2ax + a^2 - b^2 = 0$,

(B) $x^2 - 2ax - a^2 + b^2 = 0$,

(C) $x^2 - 2bx - a^2 + b^2 = 0$,

(D) $x^2 - 2bx + a^2 - b^2 = 0$.

7. Suppose $f(x) = 2(x^2 + \frac{1}{x^2}) - 3(x + \frac{1}{x}) - 1$ where x is real and $x \neq 0$. Then the solutions of $f(x) = 0$ are such that their product is

(A) 1,

(B) 2,

(C) -1,

(D) -2.

8. Toss a fair coin 43 times. What is the number of cases where number of 'Head'> number of 'Tail'?

(A) 2^{43},

(B) $2^{43} - 43$,

(C) 2^{42},

(D) None of the above.

9. The minimum number of real roots of $f(x) = |x|^3 + a|x|^2 + b|x| + c$, where a, b and c are real, is

(A) 0,

(B) 2,

(C) 3,

(D) 6.

10. Suppose $f(x, y)$ where x and y are real, is a differentiable function satisfying the following properties:

(i) $f(x+k, y) = f(x, y) + ky$;

(ii) $f(x, y+k) = f(x, y) + kx$; and

(iii) $f(x, 0) = m$, where m is a constant.

Then $f(x, y)$ is given by

(A) $m + xy$,

(B) $m + x + y$,

(C) mxy,

(D) None of the above.

11. Let $I = \int_2^{343} \{x - [x]\}^2 dx$ where $[x]$ denotes the largest integer less than or equal to x. Then the value of I is

(A) $\frac{343}{3}$,

(B) $\frac{343}{2}$,

(C) $\frac{341}{3}$,

(D) None of the above.

12. The coefficients of three consecutive terms in the expression of $(1+x)^n$ are 165, 330 and 462. Then the value of n is

(A) 10,

(B) 11,

(C) 12,

(D) 13.

13. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in

(A) $[\frac{1}{2}, 1]$,

(B) $[-1, 1]$,

(C) $[-\frac{1}{2}, \frac{1}{2}]$,

(D) $[-\frac{1}{2}, 1]$.

14. Let the function $f(x)$ be defined as $f(x) = |x-4| + |x-5|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at all points,

(B) $f(x)$ is differentiable at $x = 4$, but not at $x = 5$,

(C) $f(x)$ is differentiable at $x = 5$ but not at $x = 4$,

(D) None of the above.

15. The value of the integral $\int_0^1 \int_0^x x^2 e^{xy} dx dy$ is

(A) e,

(B) $\frac{e}{2}$,

(C) $\frac{1}{2}(e-1)$,

(D) $\frac{1}{2}(e-2)$.

16. Let $\mathcal{N} = \{1, 2, \ldots\}$ be a set of natural numbers. For each $x \in \mathcal{N}$, define $A_n = \{(n+1)k, k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_2,

(B) A_4,

(C) A_5,

(D) A_6.

17. $\lim_{x \to 0} \{\frac{1}{x}(\sqrt{1 + x + x^2} - 1)\}$ is

(A) 0,

(B) 1,

(C) $\frac{1}{2}$,

(D) Non-existent.

18. The value of $\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \ldots + (n+1)\binom{n}{n}$ equals

(A) $2^n + n2^{n-1}$,

(B) $2^n - n2^{n-1}$,

(C) 2^n,

(D) 2^{n+2}.

19. The rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is

(A) 1,
(B) 2,
(C) 3,
(D) 4.

20. Suppose an odd positive integer $2n+1$ is written as a sum of two integers so that their product is maximum. Then the integers are

(A) $2n$ and 1,
(B) $n+2$ and $n-1$,
(c) $2n-1$ and 2,
(D) None of the above.

21. If $|a| < 1, |b| < 1$, then the series $a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\ldots\ldots$ converges to
(A) $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$,
(B) $\frac{a(a+b)}{1-a(a+b)}$,
(C) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$,
(D) $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

22. Suppose $f(x) = x^3 - 6x^2 + 24x$. Then which of the following statements is true?
(A) $f(x)$ has a maxima but no minima,
(B) $f(x)$ has a minima but no maxima,
(C) $f(x)$ has a maxima and a minima,
(D) $f(x)$ has neither a maxima nor a minima.

23. An urn contains 5 red balls, 4 black balls and 2 white balls. A player draws 2 balls one after another with replacement. Then the probability of getting at least one red ball or at least one white ball is

(A) $\frac{105}{121}$,

(B) $\frac{67}{121}$,

(C) $\frac{20}{121}$,

(D) None of the above.

24. If $log_t x = \frac{1}{t-1}$ and $log_t y = \frac{t}{t-1}$, where $log_t x$ stands for logarithm of x to the base t. Then the relation between x and y is

(A) $y^x = x^{1/y}$,
(B) $x^{1/y} = y^{1/x}$,
(C) $x^y = y^x$,
(D) $x^y = y^{1/x}$.

25. Suppose $\frac{f'(x)}{f(x)} = 1$ for all x. Also, $f(0) = e^2$ and $f(1) = e^3$. Then $\int_{-2}^{2} f(x)dx$ equals
(A) $2e^2$,
(B) $e^2 - e^{-2}$,
(C) $e^4 - 1$,
(D) None of the above.

26. The minimum value of the objective function $z = 5x + 7y$, where $x \geq 0$ and $y \geq 0$, subject to the constraints

$2x + 3y \geq 6, \;\; 3x - y \leq 15, \;\; -x + y \leq 4$, and $2x + 5y \leq 27$
is

(A) 14,
(B) 15,
(C) 25,
(D) 28.

27. Suppose A is a 2×2 matrix given as $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$.
Then the matrix $A^2 - 3A - 13I$, where I is the 2×2 identity matrix, equals

(A) I,
(B) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$,
(C) $\begin{pmatrix} 1 & 5 \\ 3 & 0 \end{pmatrix}$,
(D) None of the above.

28. The number of permutations of the letters $a, \; b, \; c,$ and d such that b does not follow a, c does not follow b, and d does not follow c is
(A) 14,
(B) 13,
(C) 12,
(D) 11.

29. Given n observations $x_1, \; x_2, \; \ldots, x_n$, which of the following statements is true ?
(A) The mean deviation about arithmetic mean can exceed the standard deviation,
(B) The mean deviation about arithmetic mean cannot exceed the standard deviation,

(C) The root mean square deviation about a point A is least when A is the median,

(D) The mean deviation about a point A is minimum when A is the arithmetic mean.

30. Consider the following classical linear regression of y on x,

$$y_i = \beta x_i + u_i, i = 1, 2, \ldots, \ldots, n$$

where $E(u_i) = 0$, $V(u_i) = \sigma^2$ for all i, and $u_i's$ are homoscedastic and non-autocorrelated. Now, let $\hat{u}_i$ be the ordinary least square estimate of u_i. Then which of the following statements is true?

(A) $\sum_{i=1}^{n} \hat{u}_i = 0$,
(B) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i = 0$,
(C) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i \neq 0$,
(D) $\sum_{i=1}^{n} x_i \hat{u}_i = 0$.

Syllabus for PEB (Economics), 2013

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for PEB (Economics), 2013

1. An agent earns w units of wage while young, and earns nothing while old. The agent lives for two periods and consumes in both the periods. The utility function for the agent is given by $u = \log c_1 + \log c_2$, where c_i is the consumption in period $i = 1, 2$. The agent faces a constant rate of interest r (net interest rate) at which it can freely lend or borrow.

   (a) Find out the level of saving of the agent while young.

   (b) What would be the consequence of a rise in the interest rate, r, on the savings of the agent?

2. Consider a city that has a number of fast food stalls selling Masala Dosa (MD). All vendors have a marginal cost of Rs. 15/- per MD, and can sell at most 100 MD a day.

   (a) If the price of an MD is Rs. 20/-, how much does each vendor want to sell?

   (b) If demand for MD be $d(p) = 4400 - 120p$, where p denotes price per MD, and each vendor sells exactly 100 units of MD, then how many vendors selling MD are there in the market?

   (c) Suppose that the city authorities decide to restrict the number of vendors to 20. What would be the market price of MD in that case?

   (d) If the city authorities decide to issue permits to the vendors keeping the number unchanged at 20, what is the maximum that a vendor will be willing to pay for obtaining such a permit?

3. A firm is deciding whether to hire a worker for a day at a daily wage of Rs. 20/-. If hired, the worker can work for a maximum of 10 hours during the day. The worker can be used to produce two intermediate inputs, 1 and 2, which can then be combined to produce a final good. If

the worker produces only 1, then he can produce 10 units of input 1 in an hour. However, if the worker produces only 2, then he can produce 20 units of input 2 in an hour. Denoting the levels of production of the amount produced of the intermediate goods by k_1 and k_2 , the production function of the final good is given by $\sqrt{k_1 k_2}$. Let the final product be sold at the end of the day at a per unit price of Rs. 1/-. Solve for the firms optimal hiring, production and sale decision.

4. A monopolist has contracted with the government to sell as much of its output as it likes to the government at Rs. 100/- per unit. Its sales to the government are positive, and it also sells its output to buyers at Rs. 150/- per unit. What is the price elasticity of demand for the monopolists services in the private market?

5. Consider the following production function with usual notations.

$$Y = K^{\alpha}L^{1-\alpha} - \beta K + \theta L \text{ with } 0 < \alpha < 1, \beta > 0, \theta > 0.$$

Examine the validity of the following statements.

(a) Production function satisfies constant returns to scale.

(b) The demand function for labour is defined for all non-negative wage rates.

(c) The demand function for capital is undefined when price of capital service is zero.

6. Suppose that due to technological progress labour requirement per unit of output is halved in a Simple Keynesian model where output is proportional to the level of employment. What happens to the equilibrium level of output and the equilibrium level of employment in this case? Consider a modified Keynesian model where consumption expenditure is proportional to labour income and wage-rate is given. Does technological progress produce a different effect on the equilibrium level of output in this case?

7. A positive investment multiplier does not exist in an open economy simple Keynesian model when the entire amount of investment goods is supplied from import. Examine the validity of this statement.

8. A consumer consumes two goods, x_1 and x_2, with the following utility function

$$U(x_1, x_2) = U_1(x_1) + U_2(x_2).$$

Suppose that the income elasticity is positive. It is claimed that in the above set-up all goods are normal. Prove or disprove this claim.

9. A consumer derives his market demand, say x, for the product X as $x = 10 + \frac{m}{10p_x}$, where $m > 0$ is his money income and p_x is the price per unit of X. Suppose that initially he has money income $m = 120$, and the price of the product is $p_x = 3$. Further, the price of the product is now changed to $p'_x = 2$. Find the price effect. Then decompose price effect into substitution effect and income effect.

10. Consider an otherwise identical Solow model of economic growth where the entire income is consumed.

    (a) Analyse how wage and rental rate on capital would change over time.

    (b) Can the economy attain steady state equilibrium?

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2014

Syllabus for PEA (Mathematics), 2014

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).
Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.
Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.
Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample questions for PEA (Mathematics), 2014

1. Let $f(x) = \frac{1-x}{1+x}$, $x \neq -1$. Then $f(f(\frac{1}{x}))$, $x \neq 0$ and $x \neq -1$, is

   (a) 1,

   (b) x,

   (c) x^2,

   (d) $\frac{1}{x}$.

2. What is the value of the following definite integral?

$$2\int_0^{\frac{\pi}{2}} e^x \cos(x)dx.$$

   (a) $e^{\frac{\pi}{2}}$.

   (b) $e^{\frac{\pi}{2}} - 1$.

   (c) 0.

(d) $e^{\frac{\pi}{2}} + 1$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as follows:

$$f(x) = |x - 1| + (x - 1).$$

Which of the following is not true for f?

(a) $f(x) = f(x')$ for all $x, x' < 1$.

(b) $f(x) = 2f(1)$ for all $x > 1$.

(c) f is not differentiable at 1.

(d) The derivative of f at $x = 2$ is 2.

4. Population of a city is 40 % male and 60 % female. Suppose also that 50 % of males and 30 % of females in the city smoke. The probability that a smoker in the city is male is closest to

(a) 0.5.

(b) 0.46.

(c) 0.53.

(d) 0.7.

5. A blue and a red die are thrown simultaneously. We define three events as follows:

- Event E: the sum of the numbers on the two dice is 7.
- Event F: the number on the blue die equals 4.
- Event G: the number on the red die equals 3.

Which of the following statements is true?

(a) E and F are disjoint events.

(b) E and F are independent events.

(c) E and F are not independent events.

(d) Probability of E is more than the probability of F.

6. Let $p > 2$ be a prime number. Consider the following set containing 2×2 matrices of integers:

$$T_p = \left\{ A = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \{0, 1, \ldots, p-1\} \right\}.$$

A matrix $A \in T_p$ is p-special if determinant of A is not divisible by p. How many matrices in T_p are p-special?

(a) $(p-1)^2$.

(b) $2p - 1$.

(c) p^2.

(d) $p^2 - p + 1$.

7. A "good" word is any seven letter word consisting of letters from $\{A, B, C\}$ (some letters may be absent and some letter can be present more than once), with the restriction that A cannot be followed by B, B cannot be followed by C, and C cannot be followed by A. How many good words are there?

(a) 192.

(b) 128.

(c) 96.

(d) 64.

8. Let n be a positive integer and $0 < a < b < \infty$. The total number of real roots of the equation $(x-a)^{2n+1} + (x-b)^{2n+1} = 0$ is

(a) 1.

(b) 3.

(c) $2n - 1$.

(d) $2n + 1$.

9. Consider the optimization problem below:

$$\max_{x,y} x + y$$
$$\text{subject to } 2x + y \leq 14$$
$$-x + 2y \leq 8$$
$$2x - y \leq 10$$
$$x, y \geq 0.$$

The value of the objective function at optimal solution of this optimization problem:

(a) does not exist.

(b) is 8.

(c) is 10.

(d) is unbounded.

10. A random variable X is distributed in $[0, 1]$. Mr. Fox believes that X follows a distribution with cumulative density function (cdf) $F : [0, 1] \to [0, 1]$ and Mr. Goat believes that X follows a distribution with cdf $G : [0, 1] \to [0, 1]$. Assume F and G are differentiable, $F \neq G$ and $F(x) \leq G(x)$ for all $x \in [0, 1]$. Let $\mathbb{E}_F[X]$ and $\mathbb{E}_G[X]$ be the expected values of X for Mr. Fox and Mr. Goat respectively. Which of the following is true?

(a) $\mathbb{E}_F[X] \leq \mathbb{E}_G[X]$.

(b) $\mathbb{E}_F[X] \geq \mathbb{E}_G[X]$.

(c) $\mathbb{E}_F[X] = \mathbb{E}_G[X]$.

(d) None of the above.

11. Let $f : [0,2] \to [0,1]$ be a function defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \leq \alpha \\ \frac{1}{2} & \text{if } x \in (\alpha, 2]. \end{cases}$$

where $\alpha \in (0,2)$. Suppose X is a random variable distributed in $[0,2]$ with probability density function f. What is the probability that the realized value of X is greater than 1?

(a) 1.

(b) 0.

(c) $\frac{1}{2}$.

(d) $\frac{3}{4}$.

12. The value of the expression

$$\sum_{k=1}^{100} \int_0^1 \frac{x^k}{k} dx$$

is

(a) $\frac{100}{101}$.

(b) $\frac{1}{99}$.

(c) 1.

(d) $\frac{99}{100}$.

13. Consider the following system of inequalities.

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_2 - x_3 &\leq -2 \\ x_3 - x_4 &\leq 10 \\ x_4 - x_2 &\leq \alpha \\ x_4 - x_3 &\leq -4, \end{aligned}$$

where α is a real number. A value of α for which this system has a solution is

(a) -16.

(b) -12.

(c) -10.

(d) None of the above.

14. A fair coin is tossed infinite number of times. The probability that a head turns up for the first time after even number of tosses is

(a) $\frac{1}{3}$.

(b) $\frac{1}{2}$.

(c) $\frac{2}{3}$.

(d) $\frac{3}{4}$.

15. An entrance examination has 10 "true-false" questions. A student answers all the questions randomly and his probability of choosing the correct answer is 0.5. Each correct answer fetches a score of 1 to the student, while each incorrect answer fetches a score of zero. What is the probability that the student gets the mean score?

(a) $\frac{1}{4}$.

(b) $\frac{63}{256}$.

(c) $\frac{1}{2}$.

(d) $\frac{1}{8}$.

16. For any positive integer k, let S_k denote the sum of the infinite geometric progression whose first term is $\frac{(k-1)}{k!}$ and common ratio is $\frac{1}{k}$. The value of the expression $\sum_{k=1}^{\infty} S_k$ is

(a) e.

(b) $1+e$.

(c) $2+e$.

(d) e^2.

17. Let $G(x) = \int_0^x te^t dt$ for all non-negative real number x. What is the value of $\lim_{x\to 0} \frac{1}{x}G'(x)$, where $G'(x)$ is the derivative of G at x.

    (a) 0.

    (b) 1.

    (c) e.

    (d) None of the above.

18. Let $\alpha \in (0,1)$ and $f(x) = x^\alpha + (1-x)^\alpha$ for all $x \in [0,1]$. Then the maximum value of f is

    (a) 1.

    (b) greater than 2.

    (c) in $(1,2)$.

    (d) 2.

19. Let n be a positive integer. What is the value of the expression

$$\sum_{k=1}^{n} kC(n,k),$$

where $C(n,k)$ denotes the number of ways to choose k out of n objects?

    (a) $n2^{n-1}$.

    (b) $n2^{n-2}$.

    (c) 2^n.

    (d) $n2^n$.

20. The first term of an arithmetic progression is a and common difference is $d \in (0,1)$. Suppose the k-th term of this arithmetic progression equals the sum of the infinite geometric progression whose first term is a and common ratio is d. If $a > 2$ is a prime number, then which of the following is a possible value of d?

(a) $\frac{1}{2}$.
(b) $\frac{1}{3}$.
(c) $\frac{1}{5}$.
(d) $\frac{1}{9}$.

21. In period 1, a chicken gives birth to 2 chickens (so, there are three chickens after period 1). In period 2, each chicken born in period 1 either gives birth to 2 chickens or does not give birth to any chicken. If a chicken does not give birth to any chicken in a period, it does not give birth in any other subsequent periods. Continuing in this manner, in period $(k + 1)$, a chicken born in period k either gives birth to 2 chickens or does not give birth to any chicken. This process is repeated for T periods - assume no chicken dies. After T periods, there are in total 31 chickens. The maximum and the minimum possible values of T are respectively

(a) 12 and 4.
(b) 15 and 4.
(c) 15 and 5.
(d) 12 and 5.

22. Let a and p be positive integers. Consider the following matrix

$$A = \begin{bmatrix} p & 1 & 1 \\ 0 & p & a \\ 0 & a & 2 \end{bmatrix}$$

If determinant of A is 0, then a possible value of p is

(a) 1.
(b) 2.
(c) 4.
(d) None of the above.

23. For what value of α does the equation $(x-1)(x^2-7x+\alpha)=0$ have exactly two unique roots?

    (a) 6.
    (b) 10.
    (c) 12.
    (d) None of the above.

24. What is the value of the following infinite series?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \log_e 3^k.$$

    (a) $\log_e 2$.
    (b) $\log_e 2 \log_e 3$.
    (c) $\log_e 6$.
    (d) $\log_e 5$.

25. There are 20 persons at a party. Each person shakes hands with some of the persons at the party. Let K be the number of persons who shook hands with odd number of persons. What is a possible value of K?

    (a) 19.
    (b) 1.
    (c) 20.
    (d) All of the above.

26. Two independent random variables X and Y are uniformly distributed in the interval $[0,1]$. For a $z \in [0,1]$, we are told that probability that $\max(X,Y) \leq z$ is equal to the probability that $\min(X,Y) \leq (1-z)$. What is the value of z?

    (a) $\frac{1}{2}$.

x

(b) $\frac{1}{\sqrt{2}}$.

(c) any value in $[\frac{1}{2}, 1]$.

(d) None of the above.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies for all $x, y \in \mathbb{R}$

$$f(x+y)f(x-y) = \big(f(x) + f(y)\big)^2 - 4x^2 f(y).$$

Which of the following is not possible for f?

(a) $f(0) = 0$.

(b) $f(3) = 9$.

(c) $f(5) = 0$.

(d) $f(2) = 2$.

28. Consider the following function $f : \mathbb{R} \to \mathbb{Z}$, where $\mathbb{R}$ is the set of all real numbers and $\mathbb{Z}$ is the set of all integers.

$$f(x) = \lceil x \rceil,$$

where $\lceil x \rceil$ is the smallest integer that is larger than x. Now, define a new function g as follows. For any $x \in \mathbb{R}$, $g(x) = |f(x)| - f(|x|)$, where $|x|$ gives the absolute value of x. What is the range of g?

(a) $\{0, 1\}$.

(b) $[-1, 1]$.

(c) $\{-1, 0, 1\}$.

(d) $\{-1, 0\}$.

29. The value of $\lim_{x \to -1} \frac{x+1}{|x+1|}$ is.

(a) 1.

(b) -1.

(c) 0.

(d) None of the above.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = 2$ if $x \leq 2$ and $f(x) = a^2 - 3a$ if $x > 2$, where a is a positive integer. Which of the following is true?

   (a) f is continuous everywhere for some value of a.

   (b) f is not continuous.

   (c) f is differentiable at $x = 2$.

   (d) None of the above.

Syllabus for PEB (Economics), 2014

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.
Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2014

1. Consider a firm that can sell in the domestic market where it is a monopolist, and/or in the export market. The domestic demand is given by $p = 10 - q$, and export price is 5. Suppose the firm has a constant marginal cost of 4 and a capacity constraint on output of 100.

   (a) Solve for the optimal production plan of the firm. [**15 marks**]

   (b) Solve for the optimal production plan of the firm if its constant marginal cost is 6. [**5 marks**]

2. (a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

   $$-\left[(10-a)^2 + (10-b)^2\right].$$

   Suppose prices of both the goods are equal to 1.

   i. Solve for the optimal consumption of the consumer when his income is 40. [**10 marks**]

   ii. What happens to his optimal consumption when his income goes down to 10. [**5 marks**]

   (b) A monopolist faces the demand curve $q = 60 - p$ where p is measured in rupees per unit and q in thousands of units. The monopolist's total cost of production is given by $C = \frac{1}{2}q^2$.

i. What is the deadweight loss due to monopoly? [**3 marks**]

ii. Suppose a government could set a price ceiling (maximum price) that the monopolist can charge. Find the price ceiling that the government should set to minimize the deadweight loss. [**2 marks**]

3. (a) A cinema hall has a capacity of 150 seats. The owner can offer students a discount on the price when they show their student IDs. The demand for tickets from students is

$$D_s = 220 - 40P_s,$$

where P_s is the price of tickets for students after the discount. The demand for tickets for non-students is

$$D_n = 140 - 20P_n,$$

where P_n is the price of tickets for non-students.

i. What is the maximum profit the owner can make? [**8 marks**]

ii. What is the maximum profit he could make if the demand functions of students and non-students were interchanged? [**4 marks**]

(b) There are 11 traders and 6 identical (indivisible) chickens. Each trader wants to consume at most one chicken. There is also a (divisible) good called "money". Let D_i equal to 1 indicate that trader i consumes a chicken; 0 if he does not. Trader $i's$ utility function is given by $u_iD_i + m_i$, where u_i is the value he attaches to consuming a chicken, m_i is the units of money that the trader has. The valuations for the 11 traders are:

$u_1 = 10$; $u_2 = 8$; $u_3 = 7$; $u_4 = 4$; $u_5 = 3$; $u_6 = 1$; $u_7 = u_8 = 3$; $u_9 = 5$; $u_{10} = 6$; $u_{11} = 8$.

Initially each trader is endowed with 25 units of money. Traders $6, 7, 8, 9, 10, 11$ are endowed with one chicken each.

i. What is a possible equilibirum market price (units of money per chicken) in a competitive market? [**4 marks**]

ii. Is the equilibrium unique? [**4 marks**]

4. (a) Consider a monopolist who faces a market demand for his product:

$$p(q) = 20 - q,$$

where p is the price and q is the quantity. He has a production function given by

$$q = \min\left\{\frac{L}{2}, \frac{K}{3}\right\},$$

where L denotes labour and K denotes capital. There is a physical restriction on the availability of capital, that is, $\overline{K}$. Let both wage rates (w) and rental rates (r) be equal to 1. Find the monopoloy equilibrium quantity and price when (i) when $\overline{K} = 24$; (ii) $\overline{K} = 18$. [**12 marks**]

(b) Define Samuelson's Weak Axiom of Revealed Preference (WARP). [**2 marks**]

(c) Prove that WARP implies non-positivity of the own-price substitution effect and the demand theorem. [**6 marks**]

5. Consider two firms: 1 and 2, with their output levels denoted by q_1 and q_2. Suppose both have identical and linear cost functions, $C(q_i) = q_i$. Let the market demand function be $q = 10 - p$, where q denotes aggregate output and p the market price.

(a) Suppose the firms simultaneously decide on their output levels. Define the equilibrium in this market. Solve for the reaction functions of the two fims. Using these, find the equilibrium. [**10 marks**]

(b) Suppose the firms still compete over quantities, but both have a capacity constraint at an output level of 2. Find these reaction functions and the equilibrium in this case. [**10 marks**]

6. (a) Suppose the government subsidizes housing expenditures of low-income families by providing them a rupee-for-rupee subsidy for

their expenditure. The Lal family qualifies for this subsidy. They spend Rs. 250 on housing, and receive Rs. 250 as subsidy from the government.

Recently, a new policy has been proposed to replace the earlier policy. The new policy proposes to provide each low income family with a lump-sum transfer of Rs. 250, which can be used for housing or other goods.

i. Explain graphically if the Lal family would prefer the current program over the proposed program. [**6 marks**]

ii. Can they be indifferent between the two programs? [**3 marks**]

iii. Does the optimal consumption of housing and other goods change compared to the subsidy scheme? If yes, how? [**3 marks**]

(b) A drug company company is a monopoly supplier of Drug X which is protected by a patent. The demand for the drug is

$$p = 100 - X$$

and the monopolist's cost function is

$$C = 25 + X^2$$

i. Determine the profit maximizing price and quantity of the monopolist. [**2 marks**]

ii. Suppose the patent expires at a certain point in time, and after that any new drug company can enter the market and produce Drug X, facing the same cost function. What will be the competitive equilibrium industry output and price? How many firms will be there in the market? [**6 marks**]

7. Assume that an economy's production function is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t is output at time t, K_t is the capital stock at time t and N is the *fixed* level of employment (number of workers), $\alpha \in (0, 1)$ is the share of output paid to capital. The evolution of the capital stock is given by

$$K_{t+1} = (1-\delta)\, K_t + I_t$$

where I_t is investment at time t and $\delta \in [0,1]$ is the depreciation rate.

(a) Derive an expression for $\frac{Y}{N}$. [**5 marks**]

(b) How large is the effect of an increase in the savings rate on the steady state level of output per worker. [**10 marks**]

(c) What is the savings rate that would maximize steady state consumption per worker? [**5 marks**]

8. In an IS-LM model, graphically compare the effect of an expansionary monetary policy with an expansionary fiscal policy on investment (I) in (1) the short-run and (2) the medium run (where the aggregate supply and aggregate demand curves adjust). Assume that

$$I = I(i, Y),$$

where i is the interest rate and Y is the output. Also, $\frac{\partial I}{\partial i} < 0$ and $\frac{\partial I}{\partial Y} > 0$. [**15 marks**]

Under which policy (expansionary monetary or fiscal), is the investment higher in the medium run? [**5 marks**]

9. Suppose the economy is characterized by the following equations:

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y,$$

where C is consumption, Y is the income, Y_D is the disposable income, T is tax, I is investment, and c_0, c_1, b_0, b_1 are positive constants with $c_1 < 1, b_1 < 1$. Government spending is constant.

(a) Solve for equilibrium output. [**5 marks**]

(b) What is the value of the multiplier? For the multiplier to be positive, what condition must $c_1 + b_1$ satisfy? [**5 marks**]

(c) How will equilibrium output be affected when b_0 is changed? What will happen to saving? [**5 marks**]

(d) Instead of fixed T, suppose $T = t_0 + t_1 Y$, where $t_0 > 0$ and $t_1 \in (0,1)$. What is the effect of increase in b_0 on equilibrium Y? Is it larger or smaller than the case where taxes are autonomous? [**5 marks**]

10. Consider an economy where a representitive agent lives for three periods. In the first period, she is young - this is the time when she gets education. In the second period, she is middle-aged and with the level of education acquired in the first period, she generates income. More specifically, if she has h units of education in the first period, she can earn $\overline{w}h$, in the second period, where $\overline{w}$ is the exogenously given wage rate.

The agent borrows funds for her education when she is young and repays with interest when she is middle aged. If in the first period, the agent borrows e, then the human capital h at the beginning of the second period becomes $h(e)$, where $\frac{dh}{de} > 0$ along with $\frac{d^2h}{de^2} < 0$.

In the third period of her life, she consumes out of her savings made in the second period, that is, when she was middle aged. Assume that the exogenous rate of interest (gross) on saving or borrowing is $\overline{R}$. For simplicity, assume that an agent does not consume when she is young and, thus, the life time utility is $u(c^M) + \beta u(c^O)$, where c^M and c^O are the level of consumption when they are middle-aged and old respectively and $\beta \in (0,1)$ is the discount factor.

(a) Write down the utility maximization problem of the agent and the first order conditions. [**10 marks**]

(b) How does the optimal level of education vary with the wage rate and the rate of interest? [**10 marks**]

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2015
Syllabus for PEA (Mathematics), 2015

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants. Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Sample questions for PEA (Mathematics), 2015

1. $\lim_{x\to 0+} \frac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimalpart of x, is

   (a) 0 (b) 1 (c) non-existent (d)none of these

2. $f:[0,\ 1]\to[0,\ 1]$ is continuous.Then it is true that

   (a) $f(0)=0,\ f(1)=1$

   (b) f is differentiable only at $x=\frac{1}{2}$

   (c) $f'(x)$ is constant for all $x \in (0,1)$

   (d) $f(x) = x$ for at least one $x \in [0,1]$

3. $f(x)=|x-2|+|x-4|$.Then f is

   (a) continuously differentiable at x = 2
   (b) differentiable but not continuously differentiable at $x = 2$
   (c) f has both left and right derivatives at $x = 2$
   (d) none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry.Then the number of students who passed in exactly one subject is

(a) 30 (b) 25 (c) 10 (d) none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if

(a) $a = b = c = 0$
(b) $a = c = 0$, b is any non-zero real number
(c) $a = b = 0$, c is any non-zero real number
(d) $a = 0$ and either $b = 0$ or $c = 0$

6. If the positive numbers x, y, z are in harmonic progression, then $\log(x + z) + \log(x - 2y + z)$ equals

(a) $4 \log(x - z)$ (b) $3 \log(x - z)$
(c) $2 \log(x - z)$ (d) $\log(x - z)$

7. If f (x+2y, x-2y)= xy, then f (x,y) equals

(a) $\frac{x^2 - y^2}{8}$ (b) $\frac{x^2 - y^2}{4}$ (c) $\frac{x^2 + y^2}{4}$ (d) none of these

8. The domain of the function $f(x) = \sqrt{x^2 - 1} - \log\left(\sqrt{1 - x}\right)$, $x \geq 0$, is

(a) $(-\infty, -1)$ (b) $(-1, 0)$ (c) null set (d) none of these

9. The graph of the function $y = \log(1 - 2x + x^2)$ intersects the x axis at

(a) 0, 2 (b) 0,- 2 (c) 2 (d) 0

10. The sum of two positive integers is 100.The minimum value of the sum of their reciprocals is

(a) $\frac{3}{25}$ (b) $\frac{6}{25}$ (c) $\frac{1}{25}$ (d) none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $(7, \infty)$ (d) $[7, \infty)$

12. The function $f : R \to R$ satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$, where R is the real line, and $f(1) = 7$. Then $\sum_{r=1}^{n} f(r)$ equals

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $\frac{7n(n+1)}{2}$ (d) $7n(n+1)$

13. Let f and g be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$. Suppose that for all $x \in (0, 1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

(a) 1 (b) 3 (c) -2 (d) -1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on [1, 2]. Then it is always possible to get a $k \in (1, 2)$ such that

(a) $k = 2a + b$ (b) $k = a + 2b$ (c) $k = 3a + b$ (d) none of these

15. In a sequence the first term is $\frac{1}{3}$. The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500th term is

(a) $\frac{1}{503}$ (b) $\frac{1}{501}$ (c) $\frac{1}{502}$ (d) none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

(a) 6 (b) 18 (c) 27 (d) 36

17. Let a be a positive integer greater than 2.The number of values of x for which $\int_a^x (x+y)dy = 0$ holds is

(a) 1 (b) 2 (c) a (d) a + 1

18. Let (x^*, y^*) be a solution to any optimization problem $max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \le c_1$. Let (x', y') be a solution to the same optimization problem $max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \le c_1$ with an added constraint that $g_2(x,y) \le c_2$.Then which one of the following statements is always true?

(a) $f(x^*, y^*) \ge f(x', y')$ (b) $f(x^*, y^*) \le f(x', y')$
(c) $|f(x^*, y^*)| \ge |f(x', y')|$ (d) $|f(x^*, y^*)| \le |f(x', y')|$

19. Let (x^*, y^*) be a real solution to: $max_{(x,y)\in\Re^2} \sqrt{x} + y$ subject to $px + y \le m$, where $m > 0, p > 0$ and $y^* \in (0, m)$.Then which one of the following statements is true?

(a) x^* depends only on p (b) x^* depends only on m
(c) x^* depends on both p and m (d) x^* is independent of both p and m.

20. Let $0 < a_1 < a_2 < 1$ and let $f(x; a_1, a_2) = -|x - a_1| - |x - a_2|$.Let X be the set of all values of x for which $f(x; a_1, a_2)$ achieves its maximum. Then

(a) $X = \{x | x \in \{\frac{a_1}{2}, \frac{1+a_2}{2}\}\}$ (b) $X = \{x | x \in \{a_1, a_2\}\}$
(c) $X = \{x | x \in \{0, \frac{a_1+a_2}{2}, 1\}\}$ (d) $X = \{x | x \in [a_1, a_2]\}$.

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:

(a) P(A/B) > P(A) always (b) P(A/B) < P(A) always
(c) P(A/B) > P(B) always (d) None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:

(b) A and B are necessarily independent
(c) A and B are necessarily dependent
(d) A and B are necessarily equally likely
(e) None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).

100, 100, 111, 114, 165, 210, 225, 225, 230,
575, 1200, 1900, 2100, 2100, 2650, 3300

Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:

(a) Mean will increase by Rs. 200,000 but the median will not change
(b) Both mean and median will be increased by Rs. 200,000
(c) Mean and standard deviation will both be changed
(d) Standard deviation will be increased but the median will be unchanged

24. Let Pr(X=2)= 1. Define $\mu_{2n} = E(X-\mu)^{2n}, \mu = E(X)$. Then:

(a) μ_{2n}=2 (b) μ_{2n}=0 (c) μ_{2n}> 0 (d) None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:

(a) Mean is greater than median (b) Mean is smaller than median
(c) Mean and median are same (d) None of the above

26. Puja and Priya play a fair game (i.e. winning probability is ½ for both players) repeatedly for one rupee per game. If originally Puja has *a* rupees and Priya has *b* rupees (where a>b), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

(a) 1 (b) 0 (c) b/(a+b) (d) a/(a+b)

27. An urn contains *w* white balls and *b* black balls (*w>0*) and (*b>0*).The balls are thoroughly mixed and two are drawn, one after the other, *without* replacement. Let *Wi* denote the outcome 'white on the *i*-th draw' for *i*=1,2.Which one of the following is true?

(a) P(W2) = P(W1) = w/(w+b)
(b) P(W2) = P(W1) = (w-1)/(w+b-1)
(c) P(W1)= w/(w+b), P(W2)= (w-1)/(w+b-1)
(d) P(W1)= w/(w+b), P(W2) = {w(w-1)}/{(w-b)(w+b-1)}

28. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3, 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(a) ¾ (b) ½ (c) ¼ (d) 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y=|X|. Which of the following is true?
   (a) The variables X and Yare independent and Pearson's correlation coefficient between X and Y is 0.
   (b) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.
   (c) The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1.
   (d) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1.
30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

   (a) 5/20 (b) 25/400 (c) 10/20 (d) 7/16

Syllabus for PEB (Economics), 2015

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2015

1. Consider an agent in an economy with two goods X_1 and X_2.Suppose she has income 20.Suppose also that when she consumes amounts x_1 and x_2 of the two goods respectively, she gets utility

$$u(x_1, x_2) = 2x_1 + 32x_2 - 3{x_2}^2.$$

   (a) Suppose the prices of X_1 and X_2 are each 1.What is the agent's optimal consumption bundle? [**5 marks**]
   (b) Suppose the price of X_2 increases to 4, all else remaining the same.Which consumption bundle does the agent choose now? [**5 marks**]
   (c) How much extra income must the agent be given to compensate her for the increase in price of X_2? [**10 marks**]

2. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is ½, whereas the inverse demand function is given by: p = 1 – q. The official charge per connection is set at 0; thus, the state provides a subsidy of ½ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

   (a) Find the equilibrium bribe rate per connection and the social surplus. [**5 marks**]

   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them. [**10 marks**]

   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to *c*, 0 < c < ½.Find the range of values of c for which privatization increases consumers' surplus. [**5 marks**]

3. Suppose the borders of a state, B, coincide with the circumference of a circle of radius r > 0, and its population is distributed uniformly within its borders (so that the proportion of the population living within some region of B is simply the proportion of the state's total land mass contained in that region), with total population normalized to 1.For any resident of B, the cost of travelling a distance *d* is *kd*, with k> 0.Every resident of B is endowed with an income of 10, and is willing to spend up to this amount to consume one unit of a good, G, which is imported from outside the state at zero transport cost. The Finance Minister of B has imposed an entry tax at the rate 100*t*% on shipments of G brought into B. Thus, a unit of G costs *p(1+t)* inside the borders of B, but can be purchased for just *p* outside; *p(1+t)* < 10.Individual residents of B have to decide whether to travel beyond its borders to consume the good or to purchase it inside the state. Individuals can travel anywhere to shop and consume, but have to return to their place of origin afterwards.

(a) Find the proportion of the population of B which will evade the entry tax by shopping outside the state. [**5 marks**]

(b) Find the social welfare-maximizing tax rate. Also find the necessary and sufficient conditions for it to be identical to the revenue-maximizing tax rate. [**5 marks**]

(c) Assume that the revenue-maximizing tax rate is initially positive. Find the elasticity of tax revenue with respect to the external price of G, supposing that the Finance Minister always chooses the revenue-maximizing tax rate. [**10 marks**]

4. Suppose there are two firms, 1 and 2, each producing chocolate, at 0 marginal cost. However, one firm's product is not identical to the product of the other. The inverse demand functions are as follows:

$$p_1 = A_1 - b_{11}q_1 - b_{12}q_2, p_2 = A_2 - b_{21}q_1 - b_{22}q_2;$$

where p_1 and q_1 are respectively price obtained and quantity produced by firm 1 and p_2 and $q_2$are respectively price obtained and quantity produced by firm 2.$A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$are all positive.Assume the firms choose independently how much to produce.

(a) How much do the two firms produce, assuming both produce positive quantities? [**10 marks**]

(b) What conditions on the parameters $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are together both necessary and sufficient to ensure that both firms produce positive quantities? [**5 marks**]

(c) Under what set of conditions on these parameters does this model reduce to the standard Cournot model? [**5 marks**]

5. Suppose a firm manufactures a good with labor as the only input. Its production function is $Q = L$, where Q is output and L is total labor input employed. Suppose further that the firm is a monopolist in the product market and a monopsonist in the labor market. Workers may be male (M) or female (F); thus, $L = L_M + L_F$.Let the inverse demand function for output and the supply functions for gender-specific labor be respectively $p = A - \frac{Q}{2}$; $L_i = {w_i}^{\varepsilon_i}$, $\varepsilon_i > 0$; where p is the price received per unit of the good and w_i is the wage the firm pays to each unit of labor of gender i, $i \in \{M, F\}$.Let $\varepsilon_M \varepsilon_F = 1$.Suppose, in equilibrium, the firm is observed to hire both M and F workers, but pay M workers double the wage rate that it pays F workers.

(a) Derive the exact numerical value of the elasticity of supply of male labor. [**10 marks**]

(b) What happens to total *male* labor income as a proportion of total labor income when the output demand parameter A increases? Prove your claim. [**10 marks**]

6. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm.The firm produces a single good (Y) by means of a production function$Y = F(L); F' > 0, F'' < 0$, and maximizes profits $\Pi = PY - WL$,where P is the price of Y and W is the wage rate.The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by:

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\frac{M}{P} - d(L);$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour; with $d' > 0, d'' > 0$.The household's budget constraint is given by:

$$PC + M = WL + \Pi + \overline{M} - PT;$$

where $\overline{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G.The government's budget constraint is given by:

$$M - \overline{M} = PG - PT.$$

Goods market clearing implies: $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$). [**15 marks**]

(b) Show that everything else remaining the same, a rise in $\overline{M}$ leads to an equi-proportionate rise in P. [**5 marks**]

7. Consider the Solow growth model in continuous time, where the exogenous rate of technological progress, g, is zero. Consider an intensive form production function given by:

$$f(k) = k^4 - 6k^3 + 11k^2 - 6k; \qquad (1)$$

where $k = \frac{K}{L}$ (the capital labour ratio).

(a) Specify the assumptions made with regard to the underlying extensive form production function $F(K, L)$ in the Solow growth model, and explain which ones among these assumptions are violated by (1). [**10 marks**]

(b) Graphically show that, with a suitable value of $(n + \delta)$, where n is the population growth rate, and $\delta \in [0,1]$ is the depreciation rate on capital, there exist three steady state equilibria. [**5 marks**]

(c) Explain which of these steady state equilibria are locally unstable, and which are locally stable. Also explain whether any of these equilibria can be globally stable. [**5 marks**]

8. Consider a standard Solow model in discrete time, with the law of motion of capital is given by

$$K(t + 1) = (1 - \delta)K(t) + I(t),$$

where $I(t)$ is investment at time t and$K(t)$ is the capital stock at time t; the capital stock depreciates at the rate $\delta \in [0,1]$.Suppose output, $Y(t)$, is augmented by government spending, $G(t)$, in every period, and that the economy is closed; thus:

$$Y(t) = C(t) + I(t) + G(t),$$

where $C(t)$ is consumption at time t. Imagine that government spending is given by:

$$G(t) = \sigma Y(t),$$

where $\sigma \in [0,1]$.

(a) Suppose that: $C(t) = (\emptyset - \lambda\sigma)Y(t)$; where $\lambda \in [0,1]$.Derive the effect of higher government spending (in the form of higher σ) on the steady state equilibrium. [**10 marks**]

(b) Does a higher σ lead to a lower value of the capital stock in every period (i.e, along the entire transition path)? Prove your claim. [**10 marks**]

SYLLABUS FOR MSQE
(Program Code: MQEK and MQED) 2016

Syllabus for PEA (Mathematics), 2016

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Syllabus for PEB (Economics), 2016

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

PEA 2016 (Mathematics)

Answer all questions

1. Consider the polynomial $P(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \{1, 2, \ldots, 9\}$. If $P(10) = 5861$, then the value of c is

   (a) 1.

   (b) 2.

   (c) 6.

   (d) 5.

2. Let $A \subset \mathbb{R}$, $f : A \to \mathbb{R}$ be a twice continuously differentiable function, and $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$.

   (a) $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a *sufficient* condition for x^* to be a point of local maximum of f on A;

   (b) $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a *necessary* condition for x^* to be a point of local maximum of f on A;

   (c) $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is *necessary and sufficient* for x^* to be a point of local maximum of f on A;

   (d) $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is *neither necessary nor sufficient* for x^* to be a point of local maximum of f on A.

3. You are given five observations x_1, x_2, x_3, x_4, x_5 on a variable x, ordered from lowest to highest. Suppose x_5 is increased. Then,

   (a) The mean, median, and variance, all increase.

   (b) The median and the variance increase but the mean is unchanged.

   (c) The variance increases but the mean and the median are unchanged.

(d) None of the above.

4. Suppose the sum of coefficients in the expansion $(x+y)^n$ is 4096. The largest coefficient in the expansion is:

   (a) 924.

   (b) 1024.

   (c) 824.

   (d) 724.

5. There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. I choose a card with equal probability, then a side of that card with equal probability. If the side I choose of the card is green, what is the probability that the other side is green?

   (a) $\frac{1}{3}$.

   (b) $\frac{1}{2}$.

   (c) $\frac{2}{3}$.

   (d) $\frac{3}{4}$.

6. The value of

$$\int_0^{\frac{\pi}{2}} x \sin x dx$$

   is:

   (a) 0.

   (b) -1.

   (c) $\frac{1}{2}$.

   (d) 1

7. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} ax + b & \text{if } x \geq 0 \\ \sin 2x & \text{if } x < 0 \end{cases}$$

For what values of a and b is f continuous but *not* differentiable?

(a) $a = 2$, $b = 0$.

(b) $a = 2$, $b = 1$.

(c) $a = 1$, $b = 1$.

(d) $a = 1$, $b = 0$.

8. A student wished to regress household food consumption on household income. By mistake the student regressed household income on household food consumption and found R^2 to be 0.35. The R^2 in the correct regression of household food consumption on household income is

(a) 0.65.

(b) 0.35.

c) $1 - (.35)^2$.

(d) None of the above.

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

Which of the following statements is true?

(a) $(x = 1, y = 0)$ is a local maximum of f.

(b) $(x = 1, y = 0)$ is a local minimum of f.

(c) $(x = 1, y = 0)$ is neither a local maximum nor a local minimum of f.

(d) $(x = 0, y = 0)$ is a global maximum of f.

10. Let

$$f(x) = \frac{x + \sqrt{3}}{1 - \sqrt{3}x}$$

for all $x \neq \frac{1}{\sqrt{3}}$. What is the value of $f(f(x))$?

(a) $\frac{x-\sqrt{3}}{1+\sqrt{3}x}$.

(b) $\frac{x^2+2\sqrt{3}x+3}{1-2\sqrt{3}x+3x}$.

(c) $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

(d) $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

11. The continuous random variable X has probability density $f(x)$ where

$$f(x) = \begin{cases} a & \text{if } 0 \leq x < k \\ b & \text{if } k \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $a > b > 0$ and $0 < k < 1$. Then $E(X)$ is given by:

(a) $\frac{b(1-a)^2}{2a(a-b)}$.

(b) $\frac{1}{2}$.

(c) $\frac{a-b}{(a+b)}$.

(d) $\frac{1-2b+ab}{2(a-b)}$.

12. The set of values of x for which $x^2 - 3|x| + 2 < 0$ is given by:

(a) $\{x : x < -2\} \cup \{x : x > 1\}$.

(b) $\{x : -2 < x < -1\} \cup \{x : 1 < x < 2\}$.

(c) $\{x : x < -1\} \cup \{x : x > 2\}$.

(d) None of the above.

13. The system of linear equations

$$\begin{aligned}(4d-1)x+y+z &= 0\\ -y+z &= 0\\ (4d-1)z &= 0\end{aligned}$$

has a non-zero solution if:

(a) $d=\frac{1}{4}$.

(b) $d=0$.

(c) $d\neq\frac{1}{4}$.

(d) $d=1$.

14. Suppose F is a cumulative distribution function of a random variable x distributed in $[0,1]$ defined as follows:

$$F(x)=\begin{cases} ax+b & \text{if } x\geq a\\ x^2-x+1 & \text{otherwise}\end{cases}$$

where $a\in(0,1)$ and b is a real number. Which of the following is true?

(a) F is continuous in $(0,1)$.

(b) F is differentiable in $(0,1)$.

(c) F is not continuous at $x=a$.

(d) None of the above.

15. The solution of the optimization problem

$$\max_{x,y} \; 3xy - y^3$$
$$\text{subject to}$$
$$2x + 5y \geq 20$$
$$x - 2y = 5$$
$$x, y \geq 0.$$

is given by:

(a) $x = 19, y = 7$.

(b) $x = 45, y = 20$.

(c) $x = 15$, $y = 5$.

(d) None of the above.

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Let g be the inverse of the function f. If $f'(1) = g(1) = 1$, then $g'(1)$ equals to

(a) 0.

(b) $\frac{1}{2}$.

(c) -1.

(d) 1.

17. Consider a quadratic polynomial $P(x)$. Suppose $P(1) = -3, P(-1) = -9, P(-2) = 0$. Then, which of the following is true.

(a) $P(\frac{1}{2}) = 0$.

(b) $P(\frac{5}{2}) = 0$.

(c) $P(\frac{5}{4}) = 0$.

(d) $P(\frac{3}{4}) = 0$.

18. For any positive integers k, ℓ with $k \geq \ell$, let $C(k, \ell)$ denote the number of ways in which ℓ distinct objects can be chosen from k objects. Consider $n \geq 3$ distinct points on a circle and join every pair of points by a line segment. If we pick three of these line segments uniformly at random, what is the probability that we choose a triangle?

(a) $\frac{C(n,2)}{C(C(n,2),3)}$.

(b) $\frac{C(n,3)}{C(C(n,2),3)}$.

(c) $\frac{2}{n-1}$.

(d) $\frac{C(n,3)}{C(C(n,2),2)}$.

19. Let $X = \{(x, y) \in \mathbb{R}^2 : x + y \leq 1, 2x + \frac{y}{2} \leq 1, x \geq 0, y \geq 0\}$. Consider the optimization problem of maximizing a function $f(x) = ax + by$, where a, b are real numbers, subject to the constraint that $(x, y) \in X$. Which of the following is not an optimal value of f for any value of a and b?

(a) $x = 0, y = 1$.

(b) $x = \frac{1}{3}, y = \frac{2}{3}$.

(c) $x = \frac{1}{4}, y = \frac{1}{4}$.

(d) $x = \frac{1}{2}, y = 0$.

20. Let $F : [0, 1] \to \mathbb{R}$ be a differentiable function such that its derivative $F'(x)$ is increasing in x. Which of the following is true for every $x, y \in [0, 1]$ with $x > y$?

(a) $F(x) - F(y) = (x - y)F'(x)$.

(b) $F(x) - F(y) \geq (x - y)F'(x)$.

(c) $F(x) - F(y) \leq (x - y)F'(x)$.

(d) $F(x) - F(y) = F'(x) - F'(y)$.

21. A bag contains N balls of which a ($a < N$) are red. Two balls are drawn from the bag without replacement. Let p_1 denote the probability that the first ball is red and p_2 the probability that the second ball is red. Which of the following statements is true?

(a) $p_1 > p_2$.

(b) $p_1 < p_2$.

(c) $p_2 = \frac{a-1}{N-1}$.

(d) $p_2 = \frac{a}{N}$.

22. Let $t = x + \sqrt{x^2 + 2bx + c}$ where $b^2 > c$. Which of the following statements is true?

(a) $\frac{dx}{dt} = \frac{t-x}{t+b}$.

(b) $\frac{dx}{dt} = \frac{t+2x}{2t+b}$.

(c) $\frac{dx}{dt} = \frac{1}{2x+b}$.

(d) None of the above.

23. Let A be an $n \times n$ matrix whose entry on the i-th row and j-th column is $\min(i, j)$. The determinant of A is:

(a) n.

(b) 1.

(c) $n!$

(d) 0.

24. What is the number of non-negative integer solutions of the equation $x_1+x_2+x_3 = 10$?

(a) 66.

(b) 55.

(c) 100.

(d) None of the above.

25. The value of

$$\int_{b}^{2b} \frac{xdx}{x^2+b^2} \ ,$$

$b > 0$ is:

(a) $\frac{1}{b}$.

(b) $\ln 4b^2$.

(c) $\frac{1}{2}\ln(\frac{5}{2})$.

(d) None of the above.

26. Let f and g be functions on $\mathbb{R}^2$ defined respectively by

$$f(x,y) = \tfrac{1}{3}x^3 - \tfrac{3}{2}y^2 + 2x$$

and

$$g(x,y) = x - y.$$

Consider the problems of *maximizing* and *minimizing* f on the constraint set $C = \{(x,y) \in \mathbb{R}^2 : g(x,y) = 0\}$.

(a) f has a maximum at $(x = 1, y = 1)$, and a minimum at $(x = 2, y = 2)$.

(b) f has a maximum at $(x = 1, y = 1)$, but does not have a minimum.

(c) f has a minimum at $(x = 2, y = 2)$, but does not have a maximum.

(d) f has neither a maximum nor a minimum.

27. A particular men's competition has an unlimited number of rounds. In each round, every participant has to complete a task. The probability of a participant completing the task in a round is p. If a participant fails to complete the task in a round, he is eliminated from the competition. He participates in every round before being eliminated. The competition begins with three participants. The probability that all three participants are eliminated in the same round is:

(a) $\frac{(1-p)^3}{1-p^3}$.

(b) $\frac{1}{3}(1-p)$.

(c) $\frac{1}{p^3}$.

(d) None of the above.

28. Three married couples sit down at a round table at which there are six chairs. All of the possible seating arrangements of the six people are equally likely. The probability that each husband sits next to his wife is:

(a) $\frac{2}{15}$.

(b) $\frac{1}{3}$.

(c) $\frac{4}{15}$.

(d) None of the above.

29. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function. For every $x, y, z \in \mathbf{R}$, we know that $f(x,y) + f(y,z) + f(z,x) = 0$. Then, for every $x, y \in \mathbf{R}^2$, $f(x,y) - f(x,0) + f(y,0) =$

(a) 0.

(b) 1.

(c) -1.

(d) None of the above.

30. The minimum value of the expression below for $x > 0$ is:

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$$

(a) 1.

(b) 3.

(c) 6

(d) 12.

SYLLABUS FOR MSQE
(Program Code: MQEK and MQED) 2016

Syllabus for PEA (Mathematics), 2016

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Syllabus for PEB (Economics), 2016

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

PEB (2016)

Answer any 6 questions. All questions carry equal marks.

1. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

(i) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

(ii) What is the competitive equilibrium in this economy? Justify your answer.

(iii) Are the perfectly competitive equilibria Pareto optimal?

(iv) Now consider another economy where everything is as before, apart from individual 2's preferences, which are as follows: (a) among any two any bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy. [6]+[6]+[2]+[6]

2. Consider a monopolist who can sell in the domestic market, as well as in the export market. In the domestic market she faces a demand $p_d = 10 - q_d$, where p_d and q_d are domestic price and demand respectively. In the export market she can sell unlimited quantities at a price of 4. Suppose the monopolist has a single plant with cost function $\frac{q^2}{4}$.

(a) Solve for total output, domestic sale and exports of the monopolist.

(b) Solve for the domestic and world welfare at this equilibrium. [10]+[10]

3. A consumer consumes electricity, denoted by E, and butter, denoted by B. The per unit price of B is 1. To consume electricity the consumer has to pay a fixed charge R, and a per unit price of p. If consumption of $E \leq \frac{1}{2}$ then $p = 1$; otherwise $p = 2$. The utility function of the consumer is $3E + B$, and her income is $I > R$.

(i) Draw the consumer's budget line.

(ii) If $R = 0$ and $I = 1$, find the consumer's optimal consumption of E and B.

(iii) Consider a different pricing scheme where there is a rental charge of R and the price of E is 1 for any $X \leq 1/2$, and every *additional* unit beyond $\frac{1}{2}$ is priced at $p = 2$. Find the optimal consumption of B and E when $R = 1$ and $I = 3$. [7]+[7]+[6]

4. A monopoly publishing house publishes a magazine, earning revenue from selling the magazine, as well as by publishing advertisements. Thus $R = q.p(q) + A(q)$, where R is total revenue, q denotes quantity, $p(q)$ is the inverse demand function, and $A(q)$ is the advertising revenue. Assume that $p(q)$ is decreasing and $A(q)$ is increasing in q. The cost of production $c(q)$ is also increasing in the quantity sold. Assume all functions are twice differentiable in q.

(i) Derive the profit-maximising outcome.

(ii) Is the marginal revenue curve necessarily negatively sloped?

(iii) Can the monopolist fix the price of the magazine below the marginal cost of production?[7]+[7]+[6]

5. Consider a Solow style growth model where the production function is given by

$$Y_t = A_t F(K_t, H_t)$$

where Y_t = output of the final good, K_t is the capital stock, A_t = the level of technology, and H_t = the quantity of labor used in production (the labor force). Assume technology is equal to $A_t = A_0(1+\alpha)^t$ where $\alpha > 0$ is the growth rate of technology, A_0 is the time 0 level of technology, and $H_{t+1} = (1+n)H_t$, where $n > 0$ is the labour force growth rate. The production function is homogenous of degree 1 and satisfies the usual properties. (Assume that inputs are essential and Inada conditions hold). Assume that capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is the level of investment.

(i) Define $y_t = \frac{Y_t}{H_t}$.Show that

$$y_t = A_t f(k_t)$$

where $f(k) = F(k, 1)$.

(ii) Define $k_t = \frac{K_t}{H_t}$ and $i_t = \frac{I_t}{H_t}$. Show that

$$k_{t+1} = \frac{(1-\delta)k_t + i_t}{1+n}$$

(iii) Suppose the savings rate is given by $s_t = \sigma y_t$ where $\sigma \in [0, 1]$. Derive the condition that determines the steady state capital stock when $\alpha = 0$. How many non-zero steady states are there ?

(iv) Let $\gamma_t = \frac{k_{t+1}}{k_t}$ be the gross growth rate. Suppose $\alpha = 0$. Derive an expression for γ_t and evaluate and discuss the sign for $\frac{d\gamma_t}{dk_t}$.

(v) Let $f(k_t) = k_t^{\theta}$, $A_0 = 1$, and $\alpha > 0$. Along a balanced growth path show that $\frac{k_{t+1}}{k_t}$ and $\frac{y_{t+1}}{y_t}$ grow at the same rate. [2]+[3]+[5]+[5]+[5]

6. Consider the aggregate supply curve for an economy given by

$$P_t = P_t^e(1+\mu)F(u_t, z)$$

where P_t = actual price level at time period t, P_t^e = expected prices at time t, and the function, F, given by,

$$F(u_t, z) = 1 - \alpha u_t + z$$

captures the effects of the unemployment rate (u_t) at time t and the level of unemployment benefits (z) on the price level (through their effects on wages). Assume $\mu > 0$ denotes the monopoly markup. Assume μ and z are constant.

(i) Show that the aggregate supply curve can be transformed to be written in terms of π_t (the inflation rate) and the expected inflation rate, π_t^e, i.e. $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$, where $\pi_t = \frac{P_{t-1} - P_t}{P_t}$ and $\pi_t^e = \frac{P_{t+1}^e - P_t}{P_t}$. What is this equation called ? Briefly interpret it.

(ii) Now assume that $\pi_t^e = \theta\pi_{t-1}$ where $\theta > 0$. What is this equation called ? Re-write the equation in the above bullet and interpret when $\theta = 1$ and $\theta \neq 1$.

(iii) Let $\pi_t^e = \pi_{t-1}$ Derive the natura rate of unemployment, and express the change in the inflation rate in terms of the natural rate. Briefly interpret this equation.

(iv) How would you think about wage indexation in this model ? Does wage indexation increase the effect of unemployment on inflation? Assume $\pi_t^e = \pi_{t-1}$. [8]+[3]+[6]+[6]

7. Consider an inter-temporal choice problem in which a consumer maximises utility,

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\delta},$$

where c_i is the consumption in period i, $i = 1, 2$, and δ is the discount factor (measure of the consumer's impatience), subject to

$$c_1 + \frac{c_2}{1+\delta} = Y_1 + \frac{Y_2}{1+r} \equiv W,$$

where Y_i is the consumer's income in period $i = 1, 2$, and r is the rate of interest. Assume $c_i > 1\ \forall i$.

(i) Let $u(c_i) = log(c_i)$. Find a condition such that there is consumption smoothing.

(ii) Plot the two cases where (a) the consumer biases its consumption towards the future, and (b) where the consumer biases it consumption towards the present. Put c_2 on the vertical axis and c_1 on the horizontal axis.

(iii) Suppose there is consumption smoothing. Solve for $c_1^* = c_1(r, Y_1, Y_2)$. Interpret this equation.

(iv) Define Y_P, the permanent income, as that constant stream of income (Y_P, Y_P) which gives the same lifetime income as does the fluctuating income stream (Y_1, Y_2). What does this imply about the optimal choice of c_1, c_2, and Y_P? Interpret your result graphically. [5]+[6]+[4]+[5]

8. Consider a cake of size 1 which can be divided between two individuals, A and B. Let α (resp. β) be the amount allocated to A (resp. B), where $\alpha + \beta = 1$ and $0 \leq \alpha, \beta \leq 1$. Agents A's utility function is $u_A(\alpha) = \alpha$ and that of agent B is $u_B(\beta) = \beta$.

(i) What is the set of Pareto optimal allocations in this economy?

(ii) Suppose A is asked to cut the cake in two parts, after which B can choose which of the two segments to pick for herself, leaving the other segment for agent A. How should A cut the cake?

(iii) Suppose A is altruistic, and his utility function puts weight on what B obtains, i.e. $u_A(\alpha, \beta) = \alpha + \mu\beta$, where μ is the weight on agent B's utility. (a) If $0 < \mu < 1$, does the answer to either 8(i) or 8(ii) change? (b) What if $\mu > 1$? [5]+[5]+[10]

9. A firm uses four inputs to produce an output with a production function

$$f(x_1, x_2, x_3, x_4) = \min\{x_1, x_2\} + \min\{x_3, x_4\}.$$

(i) Suppose that 1 unit of output is to be produced and factor prices are $1, 2, 3$ and 4 for x_1, x_2, x_3 and x_4 respectively. Solve for the optimal factor demands.

(ii) Derive the cost function.

(iii) What kind of returns to scale does this technology exhibit? [6]+[8]+[6]

10. Consider an IS-LM model where the sectoral demand functions are given by

$$C = 90 + 0.75Y,$$

$$G = 30, I = 300 - 50r,$$

$$(\frac{M}{P})_d = 0.25Y - 62.5r, (\frac{M}{P})_s = 500.$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Now consider an initial situation where $Y = 2500, r = 1/5$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 2, Y = 2500)$?

[10]+[5]+[5]

2017

SAMPLE QUESTIONS: PEA (MATHEMATICS)

- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.
- You will get:

  4 marks for each **correctly** answered question,

  0 marks for each **incorrectly** answered question, and

  1 mark for each **unanswered** question.

1. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are two functions such that $f(x) = ax+b$ and $g(x) = cx+d$, then $f(g(x)) = g(f(x))$ holds if and only if

   A. $f(a) = g(c)$ B. $f(b) = g(b)$ C. $f(d) = g(b)$ D. $f(c) = g(a)$

2. A box contains 90 good and 10 defective screws. If 10 screws are drawn without replacement, the probability that none of them is defective is

   A. $\frac{^{90}C_{10}}{^{100}C_{10}}$ B. $\frac{90^{10}}{100^{10}}$ C. $\frac{^{90}P_{10}}{^{100}P_{10}}$ D. None of the above

3. The number of ordered pairs of integers (x, y) satisfying the equation

$$x^2 + 6x + y^2 = 4$$

   is

   A. 2 B. 4 C. 6 D. 8

4. The value of $\lim_{x\to-\infty} \frac{3x^2 - Sin(5x)}{x^2+2}$ is

   A. 0 B. 1 C. 2 D. 3

5. The smallest integer that produces remainders of $2, 4, 6$ and 1 when divided by $3, 5, 7$ and 11 respectively is

   A. 104 B. 1154 C. 419 D. None of the above

6. Given thirty people, the probability that among the twelve months there are six containing two birthdays each and six containing three each is

   A. $\frac{30!}{2^6 6^6} \times ^{12} C_6 \times 12^{-30}$ B. $^{12}C_6 \times ^{30} C_{12}$ C. $\frac{30!}{2^6 6^6} \times ^{12} C_6 \times \frac{1}{^{30}C_{12}}$ D. None of the above

7. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. Then the number of solutions of $|x^2 - [x]| = 1$ is

   A. 0 B. 1 C. 2 D. 3

8. Let $a, b, c \in \mathbb{R}$, $a^2 + b^2 + c^2 = 1$, and $A = ab + bc + ca$. Then

   A. $-\frac{1}{2} < A < 1$ B. $-1 < A < 1$ C. $-\frac{1}{2} < A \leq 1$ D. $-\frac{1}{2} \leq A \leq 1$

9. Two dice are rolled. If the two faces are different, the probability that one is a six is

   A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

10. The minimum value of $\frac{x^2+2}{\sqrt{x^2+1}}$ (where x is a real number) is

A. 1 B. 2 C. $\sqrt{2}$ D. None of the above

11. If $f(x) = (\frac{a+x}{b+x})^{a+b+2x}$, then $f'(0)$ equals

A. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{a+b})^{a+b}$ B. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ C. $(2\log(\frac{a}{a+b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ D. None of the above

12. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random without replacement. Then the probability that none of the balls drawn is blue is

A. 10/21 B. 11/21 C. 2/7 D. 5/7

13. The letters of the word COCHIN are permuted and all the permutations are arranged lexicographically (i.e., in alphabetical order as in an English dictionary). The number of words that appear before the word COCHIN is

A. 96 B. 360 C. 192 D. 48

14. The function $f : [0, \infty) \times [0, \infty) \to \mathbb{R}$ given by $f(x, y) = x^{1/3}y^{1/2}$ is

A. convex and quasi-convex B. convex but not quasi-convex C. quasi-convex but not convex D. neither quasi-convex nor convex

15. The function in the previous question is

A. homogeneous of degree 1 and homothetic B. homogeneous of degree 1 but not homothetic C. homothetic but not homogeneous of degree 1 D. Neither homogeneous of degree 1 nor homothetic

16. Player 1 and Player 2 both start with 100 rupees. Each round of a game consists of the following:
Both players choose a number randomly and independently from 1 to 5. If both players choose the same number, then Player 1 gives rupees 10 to Player 2. Otherwise, Player 2 gives rupees 10 to Player 1. Then the expected amount of money Player 1 will be left with after playing 10 rounds of this game is

A. 120 B. 100 C. 50 D. 160

17. Let $f : (0, 1) \to \mathbb{R}$ be a function defined by

$$f(x) = \left\{ \begin{array}{ll} x^2, & \text{if } x \text{ is rational;} \\ 2 - x^2, & \text{otherwise.} \end{array} \right\}$$

Then f is continuous at

A. no point in (0, 1) B. exactly one point in (0, 1) C. exactly two points in (0, 1) D. more than two points in (0, 1)

18. The last digit of 432^{43567} is

A. 2 B. 4 C. 6 D. 8

19. Two squares are chosen at random on a chessboard (with 64 squares). Then the probability that they have a side in common is

A. 1/9 B. 1/27 C. 1/18 D. None of the above

20. Let $f : [0,1] \to \mathbb{R}$ be a function such that $f(x) = \frac{x}{x-2}$. Then f is

A. one-one B. onto C. one-one and onto D. None of the above

21. Consider the functions $f : \mathbb{R}^2 \to \mathbb{R}^3$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x+2y, x-y, -2x+3y)$ and $g(x,y) = (x+1, y+2)$. A. Both f and g are linear transformations B. f is a linear transformation, but g is not a linear transformation C. f is not a linear transformation, but g is a linear transformation D. Neither f nor g are linear transformations

22. The ratio of boys to girls at birth in Singapore is 1.09:1 Then the proportion of Singaporean families with exactly 6 children who will have at least 3 boys is

A. 0.696 B. 0.315 C. 0.521 D. 0.455

23. Let

$$X = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$

Then

A.

$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 0 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

B.

$$X^{-1} = 1/8 \begin{pmatrix} 4 & 0 & 0 \\ -2 & 4 & 0 \\ 3 & -6 & 4 \end{pmatrix}$$

C.

$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

D. X^{-1} does not exist

24. Suppose the probability of having a girl is 1/2 and so is the probability of having a boy. Now consider a family with two children. Then the probability that both the children are girls given that at least one of them is a girl is

A. 1/4 B. 2/3 C. 1/3 D. 1/2

25. Let $U : \mathbb{R}_+ \to \mathbb{R}$ be a strictly increasing function such that $U(x) \neq -1$ for all $x \in \mathbb{R}_+$, where $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. Then the function $V : \mathbb{R}_+ \to \mathbb{R}$, defined by $V(x) = \frac{U(x)}{1+U(x)}$, is

A. necessarily strictly increasing B. necessarily strictly decreasing C. necessarily constant D. None of the above

26. A box contains three coins: two regular coins and one fake two-headed coin (i.e., $P(H) = 1$). Bagha picks a coin at random and tosses it, and gets head. Then the probability that it is the two-headed coin is

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. None of the above

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(x)f'(x) < 0$ for all $x \in \mathbb{R}$. Then
A. $f(x)$ is an increasing function B. $|f(x)|$ is an increasing function C. $f(x)$ is a decreasing function D. $|f(x)|$ is a decreasing function

28. Let X and Y be two independent discrete random variables with the CDFs F_X and F_Y. Then the CDF of $W = \min\{X, Y\}$ is

A. $F_W(w) = \frac{F_X(w)+F_Y(w)}{2}$ B. $F_W(w) = \min\{F_X(w), F_Y(w)\}$ C. $F_W(w) = F_X(w)F_Y(w)$ D. None of the above

29. Two dice are thrown simultaneously. Then the probability of getting two numbers whose product is even is

A. 1/8 B. 1/4 C. 3/4 D. None of the above

30. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. The range of the function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \frac{\sin(\pi[x])}{x^2+5}$, is

A. $(-1, 1)$ B. $[-1, 1]$ C. $\{-1, 1\}$ D. $\{0\}$

PEB (ECONOMICS) --- SAMPLE QUESTIONS 2017

For each of the thirty questions, there are four possible answers. You will get 4 marks for each correctly answered question, 1 mark for each unanswered question, and 0 marks for each incorrectly answered question.

Question 1: Under nominal wage rigidity, the short run aggregate supply schedule will be

(a) Vertical
(b) Horizontal
(c) Upward sloping
(d) Downward sloping

Question 2: Under rational expectations and no nominal rigidities, aggregate output is sensitive to ___________ supply and ___________ demand shocks.

(a) Anticipated, anticipated
(b) Anticipated, unanticipated
(c) Unanticipated, anticipated
(d) Unanticipated, unanticipated

Question 3: Keynes argued that monetary policy was ineffective during the Great Depression because

(a) IS curve was vertical and stuck at a low level of income.
(b) Both the IS and the LM curves were vertical.
(c) IS curve was continuously shifting, while the LM curve was vertical.
(d) None of the above choices is correct.

Question 4: In the basic Solow model of growth

(a) An increase in the savings rate raises the steady-state growth rate
(b) An increase in the growth rate of population lowers the steady-state growth rate
(c) An increase in the growth rate of population has no impact on the steady-state growth rate
(d) An increase in the savings rate has no impact on the steady-state growth rate.

Question 5: The money multiplier is _________ in the reserve-deposit ratio and __________in the cash-deposit ratio.

(a) Increasing; Decreasing
(b) Decreasing; Decreasing
(c) Decreasing; Increasing
(d) Increasing; Increasing

Question 6: If the IS curve is downward sloping and the LM curve is vertical, a unit increase in government expenditure results in

(a) Crowding in and higher increase in equilibrium income
(b) No crowding out and equivalent increase in the equilibrium income

(c) Partial crowding out and lower increase in equilibrium income
(d) Complete crowding out and no increase in equilibrium income

Question 7: Monetary policy is completely ineffective in raising output if

(a) The IS curve is horizontal and the LM curve is upward sloping
(b) The IS curve is vertical and the LM curve is upward sloping
(c) The IS curve is downward sloping and the LM curve is upward sloping
(d) The IS curve is downward sloping and the LM curve is vertical.

Question 8: Which of the following statements is correct? In a closed economy, fiscal policy is more effective

(a) The smaller the induced change in interest rates and smaller the responsiveness of investment to these changes.
(b) The larger the induced change in interest rates and smaller the responsiveness of investment to these changes.
(c) The smaller the induced change in interest rates and larger the responsiveness of investment to these changes.
(d) The larger the induced change in interest rates and lager the responsiveness of investment to these changes.

Question 9: Which of the following spells the most fundamental difference between the standard Solow model of growth and the standard optimal growth model?

(a) The rate of technology progress is endogenous in the former but exogenous in the latter
(b) The savings rate is exogenous in the former, but endogenous in the latter
(c) Capital utilization is exogenous in the former, but endogenous in the latter
(d) All of the above.

Question 10: Consider a simple Keynesian medel where equilibrium output is determined by aggregate demand. Investment is autonomous and a constant proportion of the income is saved. In this framework an increase in the savings propensity has the following effect:

(a) It leads to higher level of output in the new equilibrium
(b) It leads to lower level of output in the new equilibrium
(c) The level of output in the new equilibrium remains unchanged
(d) The level of output in the new equilibrium may increase or decrease depending on the degree of increase in the savings propensity.

The following pertains to Questions 11-14: Consider an agrarian economy consisting of two single membered households. The households are engaged in own cultivation using their family land, labour and capital. Each household is endowed with 1 acre of land and 1unit of labour. However the two households differ in terms of their initial capital endowments (K^R_0 and K_0^P), where R denotes the relatively richer household and P denotes the poorer household. Assume that $2<K^R_0<4$, and

$0<K_0^P<1$. The households have access to two technologies, which are specified by the following production functions:

Technology A: $Y_t=(N_tL_t)^{1/2}(K_t)^2$;

Technology B: $Y_t=(N_tL_t)^{1/2}(K_t)^{1/2}$.

Where N_t represents land (in acres), L_t represents labour, and K_t represents capital in period t.

The households choose the technology that gives them higher output(given their land, labour and capital stock) in any period t. In every period they consume half of their total income and save, and invest the rest, which adds to the next period's capital stock. Land and labour stock remain constant over time. Existing capital stock depreciates fully upon production.

Question 11: Given their initial factor endowments, the technology choices of the rich household and poor household respectively are as follows:

(A) Household R chooses technology A; household P chooses B
(B) Household R chooses technology B; household P chooses A
(C) Both households choose technology A
(D) Both households choose technology B

Question 12: In the short run, the average capital stock in the economy (K) evolves according to the following dynamic path:

(A) $dK/dt=1/4[(K^R{}_t)^{1/2}+(K^P{}_t)^2-2(K^R{}_t+K^P{}_t)]$
(B) $dK/dt=1/4[(K^R{}_t)^2+(K^P{}_t)^2-2(K^R{}_t+K^P{}_t)]$
(C) $dK/dt=1/4[(K^R{}_t)^{1/2}+(K^P{}_t)^{1/2}-2(K^R{}_t+K^P{}_t)]$
(D) $dK/dt=1/4[(K^R{}_t)^2+(K^P{}_t)^{1/2}-2(K^R{}_t+K^P{}_t)]$

Question 13: In the long run:

(A) Income of both households grows perpetually
(B) Income of household R grows perpetually while income of household P approaches a constant.
(C) Income of household P grows perpetually while income of household R approaches a constant.
(D) Income of household R grows perpetually while income of household P falls perpetually.

Question 14: If at the end of the initial time period, the households were given a choice to spend their savings in buying more land instead of investing in capital stock:

(A) Both households would have bought more land
(B) Both households would have still invested in capital
(C) Household R would have still invested in capital but household P would have bought more land

(D) Household P would have still invested in capital but household R would have bought more land

Question 15: In the Mundell-Fleming model of a small open economy with flexible exchange rates and perfect capital mobility, suppose the economy is initially in equilibrium. If lump sum taxes are increased, what happens to the equilibrium levels of the country's (i) GDP (ii) interest rate and (iii) exchange rate?

(a) (i) falls, (ii) falls,(iii) appreciates
(b) (i) and (ii) remain unchanged; (iii) depreciates
(c) (i) falls,(ii) and (iii) remain unchanged
(d) All three remain unchanged.

Question 16: Raju consumes goods 1 and 2. Raju thinks that 2 units of good 1 is always a perfect substitute for 3 units of good 2. Which of the following utility functions is the only one that would NOT represent Raju's preferences?

(a) $U(x_1,x_2)= 9x_1^2+12x_1 x_2+4x_2^2$.
(b) $U(x_1,x_2)= \min\{3x_1, 2x_2\}$
(c) $U(x_1,x_2)= 30x_1+20x_2-10,000$.
(d) More than one of the above does NOT represent Raju's preferences.

Question 17: Riya has a demand function for mango juice given by q= .02m-2p, where m is income and p is price. Riya's income is 6,000 and she initially had to pay a price of 30 per bottle of mango juice. The price of mango juice rose to 40. The substitution effect of the price change

(a) Reduced her demand by 20.
(b) Increasedher demand by 20.
(c) Reduced her demand by 8.
(d) Reduced her demand by 32.

Question18: Ankita has a utility function $U(c_1,c_2)=c_1^{1/2}+ 0.83c_2^{1/2}$, where c_1 is her consumption in period 1 and c_2 is her consumption in period 2. Her income in period 1 is 2 times as large as her income in period 2. At what interest rate will she choose to consume the same amount in period 1 as in period 2?

(a) 0.40
(b) 0.10
(c) 0.20
(d) 0

Question 19: In a crowded city long ago, the civic authorities decided that rents were too high. The long run supply function of two-room rental apartments was given by q=18+2p and the long-run demand fuction was given by q=114-4p where p was the rental rate in rupees per week. The authorities made it illegal torent an apartment for more than 10 rupees per week. To avoid a housing shortage, the authorities agreed to pay landlords enough of a subsidy to make supply equal to demand. How much would the weekly subsidy per apartment have to be to eliminate excess demand at the ceiling price?

(a) 9
(b) 15
(c) 18
(d) 36

Question 20: A firm has the production function $f(x,y)=x^{0.70}y^{-0.30}$. This firm has

(a) Decreasing returns to scale and diminishing marginal product for factor x.
(b) Increasing returns to scale and decreasing marginal product of factor x.
(c) Constant returns to scale.
(d) None of the other options are correct.

Question 21: The production function is $f(x_1 ,x_2)=x_1^{1/2}x_2^{1/2}$. If the price of factor 1 is 8 and the price of factor 2 is 16, in what proportions should the firm use factors 1 and 2 if it wants to maximize profits?

(a) $x_1=x_2.$
(b) $x_1= 0.50\ x_2$
(c) $x_1=2x_2.$
(d) We can't tell without knowing the price of output.

Question 22: The supply curve of any firm*i* in a competitive industry is $S_i(p) =p/2$. If a firm produces 3 units of output, what are its total variable costs?

(a) 18
(b) 7
(c) 9
(d) There is not enough information given to determine total variable costs.

Question 23: The demand for slops is given by the equation q=14-p. Slops can be made at zero marginal cost. But before any slops can be produced, the firm must undertake a fixed cost of 54. Since the inventor has a patent on slops, it can be a monopolist in this new industry.

(a) The firm will produce 7 units of Slops
(b) A Pareto improvement could be achieved by having the government pay the firm a subsidy of 59 and insisting that the firm offer slops at zero price.
(c) From the point of view of social efficiency, it is best that no slops be produced.
(d) None of the other options are correct.

Question 24: A price-discriminating monopolist sells in two separate markets such that goods sold in one market are never resold in the other. It charges a price of 4 in one market and a price of 8 in the other market. At these prices, the price elasticity in the first market is -1.50 and price elasticity in the second market is -0.10. Which of the following actions is sure to raise the monopolists profits?

(a) Raise p_2
(b) Raise p_1 and Lower p_2
(c) Raise both p_1 and p_2
(d) Raise p_2 and lower p_1.

Question 25: Suppose that A and B go into the wine business in a small country where wine is difficult to grow. The demand for wine is given by p=Rs 360-.02Q where p is the price and Q is the total quantity sold. The industry consists of just the two Cournot duopolists, A and B. Imports are prohibited. A has constant marginal costs of Rs 15 and B has marginal costs of Rs. 75. How much is A's output in equilibrium?

(a) 675
(b) 1,350
(c) 337.50
(d) 1012.50

Question 26: On a certain island there are only two goods, wheat and milk.The only scarce resource is land. There are 1,000 acres of land. An acre of land will produce either 16 units of milk or 37 units of wheat. Some citizens have lots of land, some have just a little bit. The citizens of the island all have utility functions of the form U(M,W)=MW. At every pareto-optimal allocation,

(a) The number of units of milk produced equals the number of units of wheat produced.
(b) Total milk production is 8,000
(c) Every consumer's marginal rate of substitution between milk and wheat -1.
(d) None of the above is true at every pareto optimal allocation.

Question 27: Kabir's utility is $U(c,d,h) = 2c+5d-d^2-2h$, where d is the number of hours per day that he spends driving around, h is the number of hours per day spent driving around by the other people in his home town and c is the amount of money he has left to spend on other stuff besides petrol and auto repairs. Petrol and auto repairs cost Rs.50 per hour of driving . All the people in Kabir's home town have the same tastes. If each citizen believes that his own driving will not affect the amount of driving done by the others, they will all drive D1 hours per day. If they all drive the same amount, they would all be best off if each drove D2 hours per day, where

(a) D1=2 and D2=1
(b) D1=D2=2
(c) D1=4 and D2=2
(d) D1=5 and D2=0

Question 28: An airport is located next to a housing development. Where X is the number of planes that land per day and Y is the number of houses in the housing development, profits of the airport are $22X-X^2$ and profits of the developer are $32Y-Y^2-XY$. Let H1 be the number of houses built if a single profit-maximizing company owns the airport and the housing development. Let H2 be the number of houses built if the airport has to pay the developer the total "damages"XY done by the planes to the developer's profits. Then

(a) H1=H2=14
(b) H1=14 and H2=16
(c) H1=16 and H2=14
(d) H1=16 and H2=15

Question 29: A town with a population of 500 has a single public good and a single private good. Everyone's utility function is Ui(Xi,Y)=Xi-64/Y, where Xi is the amount of private good consumed by i and Y is the amount of the public good. The price of the private good is Re 1 per unit. The cost of the public good to the city is Rs. 5 per unit. Everyone has an income of at least Rs. 5,000. What is the Pareto-efficient amount of the public good for the town to provide?

(a) 80 square meters
(b) 200 square meters
(c) 100 square meters
(d) None of the other options are correct.

Question 30: Akash has the utility function U(b,w)= 6b+24w and Akshey has the utility function U(b,w)=bw. If we draw an Edgeworth Box with b on the horizontal axis and w on the vertical axis and if we measure Akash's consumptions from the lower left corner of the box, then the contract curve contains

(a) A straight line running from the upper right corner of the box to the lower left.
(b) A curve that gets steeper as you move from left to right.
(c) A straight line with slope ¼ passing through the upper right corner of the box.
(d) A curve that gets flatter as you move from left to right.

ISI 2018 PEA

1. Suppose that the level of savings varies positively with the level of income and that savings is identically equal to investment. Then the IS curve:

   (a) slopes positively.

   (b) slopes negatively.

   (c) is vertical.

   (d) does not exist.

2. Consider the Solow growth model without technological progress. Suppose that the rate of growth of the labor force is 2%. Then, in the steady-state equilibrium:

   (a) per capita income grows at the rate of 2%.

   (b) per capita consumption grows at the rate of 2%.

   (c) wage per unit of labor grows at the rate of 2%.

   (d) total income grows at the rate of 2%.

3. Consider a Simple Keynesian Model for a closed economy with government. Suppose there does not exist any public sector enterprise in the economy. Income earners are divided into two groups, Group 1 and Group 2, such that the saving propensity of the former is less than that of the latter. Aggregate planned investment is an increasing function of GDP (Y). Start with an initial equilibrium situation. Now, suppose the government imposes and collects additional taxes from Group 1 and uses the tax revenue so generated to make transfer payments to Group 2. Following this:

   (a) aggregate saving in the economy remains unchanged.

   (b) aggregate saving in the economy declines.

   (c) aggregate saving in the economy rises.

   (d) aggregate saving in the economy may change either way.

4. Suppose, in an economy, the level of consumption is fixed, while the level of investment varies inversely with the rate of interest. Then the IS curve is:

   (a) positively sloped.

   (b) negatively sloped.

(c) vertical.

(d) horizontal.

5. Suppose, in an economy, the demand function for labor is given by:

$$L^d = 100 - 5w,$$

whereas the supply function for labor is given by:

$$L^s = 5w;$$

where w denotes the real wage rate. Total labor endowment in this economy is 80 units. Suppose further that the real wage rate is flexible. Then involuntary unemployment in this economy is:

(a) 30.

(b) 50.

(c) 70.

(d) 0.

6. Consider again the economy specified in Question 5. Suppose now that the real wage rate is mandated by the government to be at least 11. Then total unemployment will be:

(a) 35.

(b) 0.

(c) 30.

(d) 10.

7. Consider a macro-economy defined by the following equations:

$$M = kPy + L(r),$$
$$S(r) = I(r),$$
$$y = \bar{y},$$

where M, P, y and r represent, respectively, money supply, the price level, output and the interest rate, while k and $\bar{y}$ are positive constants. Furthermore, $S(r)$ is the savings function, $I(r)$ is the investment demand function and $L(r)$ is the speculative demand for money function, with , $S'(r) > 0$, $I'(r) < 0$ and $L'(r) < 0$. Then, an increase in M must:

(a) increase P proportionately.

(b) reduce P.

(c) increase P more than proportionately.

(d) increase P less than proportionately.

8. Two individuals, X and Y, have to share Rs. 100. The shares of X and Y are denoted by x and y respectively, $x, y \geq 0$, $x + y = 100$. Their utility functions are $U_X(x, y) = x + \left(\frac{1}{4}\right) y$ and $U_Y(x, y) = y + \left(\frac{1}{2}\right) x$. The social welfare function is $W(U_X, U_Y) = \min\{U_X, U_Y\}$. Then the social welfare maximizing allocation is:

(a) $(44, 56)$.

(b) $(48, 52)$.

(c) $(50, 50)$.

(d) $(60, 40)$.

9. Consider two consumers. They consume one private good (X) and a public good (G). Consumption of the public good depends on the sum of their simultaneously and non-cooperatively chosen contributions towards the public good out of their incomes. Thus, if g_1 and g_2 are their contributions, then the consumption of the public good is $g = g_1 + g_2$. Let the utility function of consumer i $(i = 1, 2)$ be $U_i(x_i, g) = x_i g$. The price of the private good is $p > 0$ and the income of each consumer is $M > 0$. Then the consumers' equilibrium contributions towards the public good will be:

(a) $\left(\frac{M}{2}, \frac{M}{2}\right)$.

(b) $\left(\frac{M}{3}, \frac{M}{3}\right)$.

(c) $\left(\frac{M}{4}, \frac{M}{4}\right)$.

(d) $\left(\frac{M}{p}, \frac{M}{p}\right)$.

10. Consider two firms, 1 and 2, producing a homogeneous product and competing in Cournot fashion. Both firms produce at constant marginal cost, but firm 1 has a lower marginal cost than firm 2. Specifically, firm 1 requires one unit of labour and one unit of raw material to produce one unit of output, while firm 2 requires two units of labour and one unit of raw material to produce one unit of output. There is no fixed cost. The prices of labour and material are given and the market demand for the product is

determined according to the function $q = A - bp$, where q is the quantity demanded at price p and $A, b > 0$. Now, suppose the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 1 will:

(a) increase.

(b) decrease.

(c) remain unchanged.

(d) go up or down depending on the parameters.

11. Considered again the problem in Question 10. As before, suppose that the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 2 will:

(a) increase.

(b) decrease.

(c) remain unchanged.

(d) go up or down depending on the parameters.

12. Consider a firm which initially operates only in market A as a monopolist and faces market demand $Q = 20 - p$. Given its cost function $C(Q) = \frac{1}{4}Q^2$, it charges a monopoly price P_m in this market. Now suppose that, in addition to selling as a monopolist in market A, the firm starts selling its products in a competitive market, B, at price $\bar{p} = 6$. Under this situation the firm charges P_m^* in market A. Then:

(a) $P_m^* > P_m$.

(b) $P_m^* < P_m$.

(c) $P_m^* = P_m$.

(d) given the available information we cannot say whether $P_m^* > P_m$ or $P_m^* < P_m$.

13. Two consumers, A and B, have utility functions $U_A = \min\{x_A, y_A\}$ and $U_B = x_B + y_B$,respectively. Their endowments vectors are $e_A = (100, 100)$ and $e_B = (50, 0)$. Consider a competitive equilibrium price vector (P_X, P_Y). Then,

(a) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is the unique equilibrium price vector.

(b) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is one of the many possible equilibrium price vectors.

(c) $\left(\frac{1}{5}, \frac{2}{5}\right)$ is never an equilibrium price vector.

(d) an equilibrium price vector does not exist.

14. Suppose a firm is a monopsonist in the labor market and faces separate labor supply functions for male and female workers. The labor supply function for male workers is given by $l_M = (w_M)^k$, where l_M is the amount of male labor available when the wage offered to male workers is w_M, and k is a positive constant. Analogously, the labor supply function for female workers is given by $l_F = w_F$. Male and female workers are perfect substitutes for one another. The firm produces one unit of output from each unit of labor it employs, and sells its output in a competitive market at a price of p per unit. The firm can pay male and female workers differently if it chooses to. Suppose the firm decides to pay male workers more than female workers. Then it must be the case that:

(a) $k < \frac{1}{2}$.

(b) $\frac{1}{2} \leq k < 1$.

(c) $k = 1$.

(d) $k > 1$.

15. Consider the problem in Question 14, and assume that the firm pays male workers more than female workers. Suppose further that $p > 2$. Then the firm must:

(a) hire more male workers than female workers.

(b) hire more female workers than male workers.

(c) hire identical numbers of male and female workers.

(d) hire more females than males if $2 < p \leq 4$, but more males than females if $p > 4$.

16. Consider the system of linear equations:

$$\begin{aligned}(4a-1)\,x + y + z &= 0,\\ -y + z &= 0,\\ (4a-1)\,z &= 0.\end{aligned}$$

The value of a for which this system has a non-trivial solution (i.e., a solution other than $(0, 0, 0)$) is:

(a) $\frac{1}{2}$.

(b) $\frac{1}{4}$.

(c) $\frac{3}{4}$.

(d) 1.

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex and differentiable function with $f(0) = 1$, where $\mathbb{R}$ denotes the set of real numbers. If the derivative of f at 2 is 2, then the maximum value of $f(2)$ is:

(a) 3.

(b) 5.

(c) 10.

(d) ∞.

18. Consider the equation $2x + 5y = 103$. Then how many pairs of positive integer values can (x, y) take such that $x > y$?

(a) 7.

(b) 8.

(c) 13.

(d) 14.

19. Let X be a discrete random variable with probability mass function (PMF) $f(x)$ such that

$$\begin{aligned} &f(x) > 0 \quad \text{if } x = 0, 1, ..., n, \text{ and} \\ &f(x) = 0 \quad \text{otherwise,} \end{aligned}$$

where n is a finite integer. If $Prob\,(X \geq m \mid X \leq m) = f(m)$, then the value of m is:

(a) 0.

(b) 1.

(c) $n - 1$.

(d) none of the above.

20. Consider the function $f(x) = 2ax \log_e x - ax^2$ where $a \neq 0$. Then

(a) the function has a maximum at $x = 1$.

(b) the function has a minimum at $x = 1$.

(c) the point $x = 1$ is a point of inflexion.

(d) none of the above.

21. Let $f : [0, 10] \to [10, 20]$ be a continuous and twice differentiable function such that $f(0) = 10$ and $f(10) = 20$. Suppose $|f'(x)| \leq 1$ for all $x \in [0, 10]$. Then, the value of $f''(5)$ is

(a) 0.

(b) $\frac{1}{2}$.

(c) 1.

(d) cannot be determined from the given information.

22. Consider the system of linear equations:

$$\begin{aligned} x + 2ay + az &= 0, \\ x + 3by + bz &= 0, \\ x + 4cy + cz &= 0. \end{aligned}$$

Suppose that this system has a non-zero solution. Then a, b, c

(a) are in arithmetic progression.

(b) are in geometric progression.

(c) are in harmonic progression.

(d) satisfy $2a + 3b + 4c = 0$.

23. Let a, b, c be real numbers. Consider the function $f(x_1, x_2) = \min\{a - x_1, b - x_2\}$ .Let (x_1^*, x_2^*) be the solution to the maximization problem

$$\max f(x_1, x_2) \text{ subject to } x_1 + x_2 = c.$$

Then $x_1^* - x_2^*$ equals

(a) $\frac{c+a-b}{2}$.

(b) $\frac{c+b-a}{2}$.

(c) $a-b$.

(d) $b-a$.

24. Suppose that you have 10 different books, two identical bags and a box. The bags can each contain three books and the box can contain four books. The number of ways in which you can pack all the books is

(a) $\frac{10!}{2!3!3!4!}$.

(b) $\frac{10!}{3!3!4!}$.

(c) $\frac{10!}{2!3!4!}$.

(d) none of the above.

25. Real numbers $a_1, a_2, ..., a_{99}$ form an arithmetic progression. Suppose that

$$a_2 + a_5 + a_8 + ... + a_{98} = 205.$$

Then the value of $\sum_{k=1}^{99} a_k$ is

(a) 612.

(b) 615.

(c) 618.

(d) none of the above.

26. A stone is thrown into a circular pond of radius 1 meter. Suppose the stone falls uniformly at random on the area of the pond. The expected distance of the stone from the center of the pond is

(a) $\frac{1}{3}$.

(b) $\frac{1}{2}$.

(c) $\frac{2}{3}$.

(d) $\frac{1}{\sqrt{2}}$.

27. Suppose that there are n stairs, where n is some positive integer. A person standing at the bottom wants to reach the top. The person can climb either 1 stair or 2 stairs at a time. Let T_n be the total number of ways in which the person can reach the top. For instance, $T_1 = 1$ and $T_2 = 2$. Then, which one of the following statements is true for every $n > 2$?

(a) $T_n = n$.

(b) $T_n = 2T_{n-1}$.

(c) $T_n = T_{n-1} + T_{n-2}$.

(d) $T_n = \sum_{k=1}^{n-1} T_k$.

28. Let $Y_1, Y_2, ..., Y_n$ be the income of n individuals with $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2$ for all $i = 1, 2, ...n$. These n individuals form m groups, each of size k. It is known that individuals within the same group are correlated but two individuals in different groups are always independent. Assume that when individuals are correlated, the correlation coefficient is the same for all pairs. Consider the random variable $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$. The limiting variance of $\bar{Y}$ when m is large but k is finite is

(a) 0.

(b) $\frac{1}{k}$.

(c) 1.

(d) $\frac{\sigma^2}{k}$.

29. A person makes repeated attempts to destroy a target. Attempts are made independently of each other. The probability of destroying the target in any attempt is 0.8. Given that he fails to destroy the target in the first five attempts, the probability that the target is destroyed in the 8-th attempt is

(a) 0.032.

(b) 0.064.

(c) 0.128.

(d) 0.160.

30. Let E and F be two events such that $0 < Prob\,(E) < 1$ and $Prob\,(E \mid F) + Prob\,(E \mid F^c) = 1$. Then

(a) E and F are mutually exclusive.

(b) $Prob\,(E^c \mid F) + Prob\,(E^c \mid F^c) = 1.$

(c) E and F are independent.

(d) $Prob\,(E \mid F) + Prob\,(E^c \mid F^c) = 1.$

ISI 2018 PEB

- There are 6 questions: 2 in Group A, 2 in Group B, and 2 in Group C.
- Answer **4** questions in total and **at least 1** question from each Group.
- Each question carries equal marks.
- The maximum possible score is **100**.

Group A

1. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $\frac{1}{2}$, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of $\frac{1}{2}$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

   (a) Find the equilibrium bribe rate per connection and the social surplus.

   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them.

   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to c, $0 < c < \frac{1}{2}$. Find the range of values of c for which privatization increases consumers' surplus.

2. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

   (a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

   (b) What is the competitive equilibrium in this economy? Justify your answer.

(c) Are the perfectly competitive equilibria Pareto optimal?

(d) Now consider another economy where everything is as before, apart from individual 2's preferences, which are as follows: (a) among any two bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy.

Group B

1. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good (Y) by means of a production function $Y = F(L)$, $F'(L) > 0$, $F''(L) < 0$, and maximizes profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate. The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by

$$U = \tfrac{1}{2}\ln C + \tfrac{1}{2}\ln\left(\tfrac{M}{P}\right) - d(L),$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour with $d'(L) > 0$, $d''(L) > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \overline{M} - PT,$$

where $\overline{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G. The government's budget constraint is given by:

$$M - \overline{M} = PG - PT.$$

Goods market clearing implies $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$).

(b) Show that everything else remaining the same, a rise in $\overline{M}$ leads to an equiproportionate rise in P.

2. Consider an IS-LM model where the sectoral demand functions are given by

$$\begin{aligned} C &= 90 + 0.75Y, \\ G &= 30, \\ I &= 300 - 50r, \\ \left(\frac{M}{P}\right)_d &= 0.25Y - 62.5r, \\ \left(\frac{M}{P}\right)_s &= 500. \end{aligned}$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Consider an initial situation where $Y = 2500$, $r = \frac{1}{5}$. What is the change in the level of I that must occur before there is any change in the level of Y ?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 0.2, Y = 2500)$?

Group C

1. Answer the following questions.

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \frac{|x|}{2x} \ \forall \ x \in \mathbb{R} \setminus \{0\}.$$

Can $f(0)$ be defined in a way such that f is continuous at 0? Justify your answer.

(b) Consider the following optimization problem:

$$\max_{x \in [0,\beta]} x(1 - x),$$

where $\beta \in [0, 1]$. Let x^* be an optimal solution of the above optimization problem. For what values of β will we have $x^* = \beta$?

(c) A firm is producing two products a and b. The market price (per unit) of a and b are respectively 3 and 2. The firm has resources to produce only 10 units of a and b together. Also, the quantity of a produced cannot exceed double the quantity of b produced. What is the revenue-maximizing production plan (i.e., how many units of a and b) of the firm?

2. Answer the following questions.

   (a) A slip of paper is given to person A, who marks it with either $(+)$ or $(-)$. The probability of her writing $(+)$ is $\frac{1}{3}$. Then, the slip is passed sequentially to B, C, and D. Each of them either changes the sign on the slip with probability $\frac{2}{3}$ or leaves it as it is with probability $\frac{1}{3}$.

      i. Compute the probability that the final sign is $(+)$ if A wrote $(+)$.

      ii. Compute the probability that the final sign is $(+)$ if A wrote $(-)$.

      iii. Compute the probability that A wrote $(+)$ if the final sign is $(+)$.

   (b) There are n houses on a street numbered $h_1, \ldots, h_n$. Each house can either be painted BLUE or RED.

      i. How many ways can the houses $h_1, \ldots, h_n$ be painted?

      ii. Suppose $n \geq 4$ and the houses are situated on n points on a circle. There is an additional constraint on painting the houses: exactly two houses need to be painted BLUE and they cannot be next to each other. How many ways can the houses $h_1, \ldots, h_n$ be painted under this new constraint?

      iii. How will your answer to the previous question change if the houses are located on n points on a line.

PEA
2019

1. Robinson Crusoe will live this period (period 1) and the next period (period 2) as the only inhabitant of his island completely isolated from the rest of the world. His only income is a crop of 100 coconuts that he harvests at the beginning of each period. Coconuts not consumed in the current period spoil at the rate of 20% per period. Crusoe's preference over consumption in period 1 (c_1) and consumption in period 2 (c_2) is given by the utility function $u(c_1, c_2) = \min\{5c_1, 6c_2\}$. Crusoe's utility maximizing consumption choice is given by

   (a) $c_1 = \frac{200\times 6}{11}$, $c_2 = \frac{200\times 5}{11}$.

   (b) $c_1 = 90$, $c_2 = 108$.

   (c) $c_1 = 100$, $c_2 = 100$.

   (d) none of the above.

2. The domestic supply and demand equations for a commodity in a country are as follows: Supply: $P = 50 + Q$, Demand: $P = 200 - 2Q$, where P is the price in rupees per kilogram and Q is the quantity in thousands of kilograms. The country is a small producer in the world market where the price (which will not be affected by anything done by this country) is Rs. 60 per kilogram. The government of this country introduces a "Permit Policy" which works as follows. The government issues a fixed number of Permits – each Permit allows its owner to sell exactly 100 kilograms of the commodity in this country's market. An exporter from a foreign country cannot sell this commodity in this country unless she purchases such a Permit. Suppose the government issues 300 Permits. What is the *maximum* price an exporter is willing to pay for a Permit?

   (a) Rs. 3000.

(b) Rs. 2000.

(c) Rs. 1500.

(d) Rs. 1000.

3. SeaTel provides cellular phone service in Delhi and has some monopoly power in the sense that it has its captive customer base with each customer's weekly demand being given by: $Q = 60 - P$, where Q denotes hours of cell phone calls per week and P is the price per hour. SeaTel's total cost of providing cell phone service is given by $C = 20Q$, so that the marginal cost is $MC = 20$. Suppose SeaTel offers a "*Call-As-Much-As-You-Wish*" deal: it charges only a flat *weekly access fee*, and once a customer pays the flat access fee, he/she can call as much as he/she wishes without paying any extra usage fee per hour. The *weekly access fee* that SeaTel should charge to maximize its profit is given by

(a) 1800.

(b) 1200.

(c) 800.

(d) 40.

4. A bus stop has to be located on the interval $[0, 1]$. There are three individuals located at points 0.2, 0.3 and 0.9 on the interval. If the bus stop is located at point x, then the utility of an individual located at y is $-|y - x|$, that is, the negative of the distance between the bus stop and the individual's location. A relocation of the bus stop is said to be *Pareto improving* if at least one individual is better off and no individual is worse off from the relocation. A location of the bus stop is said to be *Pareto efficient* if there does not exist any Pareto improving relocation. Then

(a) 0.5 is the only Pareto efficient location.

(b) $\frac{0.2+0.3+0.9}{3}$ is the only Pareto efficient location.

(c) Median of 0.2, 0.3 and 0.9 is the only Pareto efficient location.

(d) none of the above.

5. Consider three goods: (a) cable television, (b) a fish in international waters, and (c) a burger. Also consider four descriptions of the goods: (A) non-rival and non-excludable, (B) rival and excludable, (C) non-rival and excludable, and (D) rival and non-excludable. In what follows we match goods to possible descriptions. Choose the correct match.

(a) (a)-(A), (b)-(C), (c)-(B).

(b) (a)-(C), (b)-(D), (c)-(A).

(c) (a)-(C), (b)-(B), (c)-(A).

(d) (a)-(C), (b)-(D), (c)-(B).

6. Consider an economy consisting of three individuals – 1, 2 and 3, two goods – A and B, and a single monopoly firm that can produce both goods at zero cost. Each individual would like to buy exactly 1 unit of the goods A and B, if at all. An individual's *valuation* of a good is defined as the maximum amount she is willing to pay for one unit of the good. Individual 1's valuation of good A is Rs. 10 and that of good B is Rs. 1. Individual 2's valuation of good A is Rs. 1, and that of good B is Rs. 10. Individual 3's valuation is Rs. 7 for good A, and Rs. 7 good B. The firm can charge a single price p_A for good A, a single price p_B for good B, and a bundled price p_{AB} such that if an individual pays p_{AB} then she gets the bundle consisting of one unit each of

goods A and B. If the monopolist sets p_A, p_B and p_{AB} to maximize its profit then

(a) $p_A = 11$, $p_B = 11$, $p_{AB} = 11$.

(b) $p_A = 11$, $p_B = 11$, $p_{AB} = 14$.

(c) $p_A = 10$, $p_B = 10$, $p_{AB} = 11$.

(d) none of the above.

7. Consider a Bertrand duopoly with two firms, 1 and 2. Both firms produce the same good that has a market demand function $p = 10 - q$. The market is equally shared in case the firms charge the same price, otherwise the lower priced firm gets the entire demand. A firm must satisfy all the demand coming to it. The cost function of firm 1 is $3q_1$, that of firm 2 is $2q_2$. Suppose prices vary along the following grid, $\{0, 0.1, 0.2, \ldots\}$. The Bertrand equilibrium is given by

(a) $p_1 = 2$, $p_2 = 2$.

(b) $p_1 = 3$, $p_2 = 2$.

(c) $p_1 = 3$, $p_2 = 2.9$.

(d) $p_1 = 3$, $p_2 = 3$.

8. Consider a monopolist with a market demand function $p = 20 - q$. It is a multi-plant monopolist with two plants, plant 1 and plant 2, where the plant specific cost function of plant i , $i = 1, 2$, is

$$c_i(q_i) = \begin{cases} 2 + 4q_i, \text{ if } q_i > 0, \\ 0, \text{ otherwise.} \end{cases}$$

The optimal monopoly profit is given by

(a) 60.

(b) 64.

(c) 68.

(d) 62.

9. Consider a closed economy in which an individual's labour supply (L) to firms is determined by the amount which maximizes her utility function $u(C, L) = C^{\alpha}(1-L)^{\beta}$, where $\alpha, \beta > 0$, $\alpha + \beta < 1$, and C is consumption expenditure which is taken to be equal to wage income (wL). Then

   (a) labour supply does not depend on the wage rate w.

   (b) labour supply is directly proportional to the wage rate w.

   (c) labour supply is inversely proportional to the wage rate w.

   (d) more information is needed to derive the labour supply.

10. In the scenario described in Question 9, assume that the economy is Keynesian, that is, investment expenditure (I) is autonomous and output (Y) is determined by aggregate demand, $Y = C + I$. The aggregate production function is given by $Y = AL^{\theta}$, where $A > 0$ is a productivity parameter and $0 < \theta < 1$. [Note that the firm's employment of labour is obtained by equating the marginal product of labour to w.] Then the marginal propensity to consume is

   (a) $\frac{\alpha+\beta}{\theta}$.

   (b) $\frac{\beta}{\theta}$.

   (c) α.

   (d) θ.

11. Consider a Solow growth model (in continuous time) with a production function with labour augmenting technological change,

$Y_t = F(K_t, A_t L_t)$, where Y_t denotes output, K_t denotes the capital stock, A_t denotes the level of total factor productivity (TFP), and L_t denotes the stock of the labour force. Assume that L_t grows at the rate $n > 0$ and A_t grows at the rate $g > 0$, that is, $\frac{\dot{L}}{L} = n$ and $\frac{\dot{A}}{A} = g$, and the capital accumulation equation is given by $\dot{K} = sY_t - \delta K_t$, where $s \in [0, 1]$ is the exogenous savings rate, and $\delta \in [0, 1]$ is the depreciation rate of capital. [Note that for any variable x, $\dot{x}$ denotes $\frac{dx}{dt}$.] Define capital in efficiency units to be $Z \equiv \frac{K}{AL}$. Then the expression for $\frac{\dot{Z}}{Z}$ is given by

(a) $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (n + g)$.

(b) $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (\delta + n + g)$.

(c) $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z}$.

(d) $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - n$.

12. In the Solow growth model described in Question 11, the growth rate of Y at the steady state is given by

(a) $n + g$.

(b) $\delta + n + g$.

(c) zero.

(d) n.

13. Consider an IS-LM model where the IS curve is represented by $0.25Y = 500 + G - i$, and money demand function is given by $\frac{M}{P} = \frac{2Y}{e^i}$. The notations are standard: Y denotes output, G denotes government expenditure, i denotes the interest rate, P is the price level and e is the exponential. Suppose the government wants to increase spending and therefore the central bank decides to change the money supply accordingly such that the interest

rate remains the same in the short run. Then the change in money supply satisfies the following condition:

(a) $\frac{dM}{dG} = e$.

(b) $\frac{dM}{dG} = \frac{Ye}{M}$.

(c) $\frac{dM}{dG} = \frac{Y}{M}$.

(d) $\frac{dM}{dG} = \frac{4M}{Y}$.

14. An agent lives for two periods. Her utility from consumption in period 1 (c_1) and consumption in period 2 (c_2) is given by $u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. The agent earns incomes w_1 in period 1 and w_2 in period 2. The rate of interest is $r > 0$. The agent chooses c_1 and c_2 so as to maximize $u(c_1, c_2)$ subject to her budget constraint. Consider a *temporary* increase in income where w_1 increases but the agent does not change her expectations about w_2. Then the marginal propensity to consume of present consumption with respect to w_1, $\frac{dc_1}{dw_1}$, is given by

(a) $\frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\right)$.

(b) $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$.

(c) $\frac{1}{1+\beta}$.

(d) 1.

15. In the scenario described in Question 14, consider a *permanent* increase in income where w_1 increases and the agent expects that w_2 will also increase by the same amount. Then $\frac{dc_1}{dw_1}$ is given by

(a) $\frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\right)$.

(b) $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$.

(c) $\frac{1}{1+\beta}$.

(d) 1.

16. For what values of a are the vectors $(0,1,a),(a,1,0),(1,a,1)$ in $\mathbb{R}^3$ linearly dependent?

(a) 0.

(b) 1.

(c) 2.

(d) $\sqrt{2}$.

17. Which of the following set of vectors form a basis of $\mathbb{R}^2$?

(a) $\{(2,1)\}$.

(b) $\{(1,1),(2,2)\}$.

(c) $\{(1,1),(1,2),(2,1)\}$.

(d) $\{(1,1),(2,3)\}$.

18. If a candidate is good he is selected in MSQE examination with probability 0.9. If a candidate is bad he is selected in MSQE examination with probability 0.2. Suppose every candidate is equally likely to be good or bad. If you meet a candidate who is selected in the MSQE examination, what is the probability that he will be good?

(a) $\frac{11}{20}$.

(b) $\frac{9}{10}$.

(c) $\frac{9}{11}$.

(d) $\frac{11}{12}$.

19. Let $S_1 = \{2, 3, 4, \ldots, 9\}$. First, an integer s_1 is drawn uniformly at random from S_1. Then s_1 and all its factors are removed from S_1. Let the new set be S_2. Next an integer s_2 is drawn uniformly at random from S_2. Then s_2 and all its factors are removed from S_2. Let the new set be S_3. Finally, an integer s_3 is drawn uniformly at random from S_3. What is the probability that $s_1 = 2, s_2 = 3, s_3 = 5$?

(a) $\frac{1}{8}$.

(b) $\frac{1}{64}$.

(c) $\frac{1}{16}$.

(d) $\frac{1}{72}$.

20. Mr. A and B are independently tossing a coin. Their coins have a probability 0.25 of coming HEAD. After each of them tossed the coin twice, we see a total of 2 HEADS. What is the probability that Mr. A had exactly one HEAD?

(a) $\frac{2}{3}$.

(b) $\frac{1}{2}$.

(c) $\frac{1}{4}$.

(d) $\frac{1}{3}$.

21. Consider the following function $f : \mathbb{R} \to \mathbb{R}$.

$$f(x) = \begin{cases} x & \text{if } x \leq e \\ x(log_e x) & \text{if } x > e. \end{cases}$$

Which of the following is true for f?

(a) f is not continuous at e.

(b) f is not differentiable at e.

(c) f is neither continuous nor differentiable at e.

(d) f is continuous and differentiable at e.

22. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous and weakly increasing function such that $\int_{-1}^{1} f(x)dx = 2\int_{-1}^{1} f(-x)dx$. Suppose $f(-1) = 0$, then $f(1)$ is

(a) 0.

(b) 1.

(c) $\frac{1}{2}$.

(d) none of the above.

23. Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ be a twice continuously differentiable function. Let $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$. Consider the following two statements: (i) if $\frac{\partial^2 f}{\partial^2 x}(x^*) \leq 0$, then x^* is a point of local maximum of f; (ii) if x^* is a point of local maximum of f, then $\frac{\partial^2 f}{\partial^2 x}(x^*) < 0$. Which of the following is true?

(a) both (i) and (ii) are correct.

(b) both (i) and (ii) are incorrect.

(c) (i) is correct but (ii) is incorrect.

(d) (ii) is correct but (i) is incorrect.

24. Consider the function $f(x) = e^x$ for all $x \in \mathbb{R}$. Which of the following is true?

(a) f is quasi-convex.

(b) f is quasi-concave.

(c) f is neither quasi-convex nor quasi-concave.

(d) f is both quasi-convex and quasi-concave.

25. Consider the following matrix A.

$$A = \begin{bmatrix} x & 0 & k \\ 1 & x & k-3 \\ 0 & 1 & 1 \end{bmatrix}$$

Suppose determinant of A is zero for two distinct real values of x. What is the least positive integer value of k?

(a) 1.

(b) 9.

(c) 10.

(d) 8.

26. Define the following function on the set of all positive integers.

$$f(n) = \begin{cases} 2 \times 4 \times \ldots \times (n-3) \times (n-1) & \text{if } n \text{ is odd} \\ 1 \times 3 \times \ldots \times (n-3) \times (n-1) & \text{if } n \text{ is even.} \end{cases}$$

What is the value of $f(n+2)f(n+1)$?

(a) $n!$.

(b) $(n+1)!$.

(c) $(n+2)!$.

(d) $(n+2)(n!)$.

27. The sequence $\{x_n\}_{n\geq 0}$ is defined as follows. We set $x_0 = 1$ and $x_n = \sum_{j=0}^{n-1} x_j$ for each integer $n \geq 1$. Then the value of the expression $\sum_{j=0}^{\infty} \frac{1}{x_j}$ is equal to

(a) ∞.

(b) 2.

(c) 3.

(d) $\frac{7}{4}$.

28. For what values of p does the following quadratic equation have more than two solutions (variable in this equation is x)?

$$(p^2 - 16)x^2 - (p^2 - 4p)x + (p^2 - 5p + 4) = 0$$

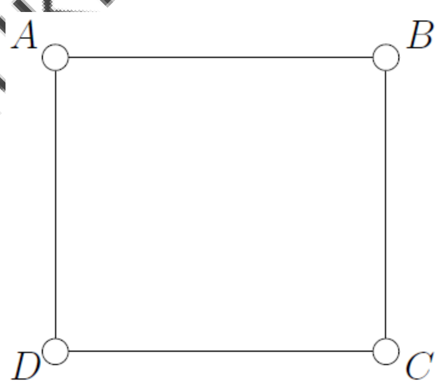
(a) No such value of p exists.

(b) -4 and 4.

(c) 1 and 4.

(d) 4.

29. Consider the square with vertices A, B, C, D as shown in the following figure. Call a pair of vertices in the square adjacent if they are connected by an edge in the figure. You have four colours: RED, BLUE, GREEN, YELLOW. How many ways can you colour the vertices A, B, C, D such that no adjacent vertices share the same colour?



(a) 84.

(b) 24.

(c) 72.

(d) 108.

30. Two players P_1 and P_2 are playing a game which involves filling the entries of an $n \times n$ matrix, where $n \geq 2$ is an **even** integer. Starting with P_1, each player takes turn to fill an unfilled entry of the matrix with a real number. The game ends when all entries are filled. Player P_1 wins if the determinant of the final matrix is non-zero. Else, player P_2 wins. A player $i \in \{1, 2\}$ has a **winning strategy** if irrespective of what the other player does, i wins by following this strategy. Which of the following is true?

    (a) Player 1 has a winning strategy.

    (b) Player 2 has a winning strategy.

    (c) No player has a winning strategy.

    (d) None of the above.

PEB
2019

Group A

1. [25 marks: $6 + 7 + 12$]

   Ms. A earns Rs. 25,000 in period 1 and Rs. 15,000 in period 2. Mr. B earns Rs. 15,000 in period 1 and Rs. 30,000 in period 2. They can borrow money at an interest rate of 200%, and can lend money at a rate of 0%. They like both consumption in period 1 (C_1) and consumption in period 2 (C_2), and their preferences are such that their chosen consumption bundles will always lie on their budget lines.

   (a) [6 marks]

   Write down the equations of their budget constraints and draw their budget lines in the same figure by plotting consumption in period 1 (C_1) (in thousand rupees) on x-axis and consumption in period 2 (C_2) (in thousand rupees) on y-axis.

   (b) [7 marks]

   Given the income profile and the market interest rates, Mr. B chooses to borrow Rs. 5,000 in period 1.

   Give an example of a consumption profile (that is, (C_1, C_2)) such that, if Ms. A chooses this profile, we would know for sure that Ms. A and Mr. B have **different preferences** for consumption in period 1 (C_1) and consumption in period 2 (C_2). Give a clear explanation for your answer.

   (c) [12 marks: $6 + 6$]

   Suppose now that Ms. A and Mr. B have the **same preferences** for C_1 and C_2, and, as in part (b), Mr. B borrows Rs. 5,000 in period 1.

i. Suppose that Ms. A chooses to be a lender in period 1. Find out, with a clear explanation, the *maximum* amount that she will lend in period 1 consistent with the fact that they have the same preferences for C_1 and C_2.

ii. Explain clearly whether Mr. B is better off than Ms. A.

2. [25 marks: $6 + 12 + 7$]

Consider an exchange economy with two agents 1 and 2 and two goods X and Y. There is one unit of both goods in the economy. An allocation is a pair $\{(X_1, Y_1), (X_2, Y_2)\}$ where $X_1 + X_2 = 1$, $Y_1 + Y_2 = 1$, and (X_1, Y_1) and (X_2, Y_2) are the consumption bundles of agents 1 and 2 respectively. The utility function for agent 1 is given by $u_1(X_1, Y_1) = X_1 \cdot Y_1$ and that of agent 2 by $u_2(X_2, Y_2) = 2X_2 + Y_2$.

(a) [6 marks]

Describe the set of Pareto-efficient allocations in the economy.

(b) [12 marks: $6 + 6$]

An allocation $\{(X_1, Y_1), (X_2, Y_2)\}$ is *envy-free* if no agent strictly prefers the consumption bundle of the other agent to her own, that is, $u_1(X_1, Y_1) \geq u_1(X_2, Y_2)$ and $u_2(X_2, Y_2) \geq u_2(X_1, Y_1)$.

i. Consider each of the two statements below. Decide whether they are true or false. Justify your answer with a proof or a counter-example as appropriate.

A. All Pareto-efficient allocations are envy-free.

B. All envy-free allocations are Pareto-efficient.

ii. Describe the set of envy-free allocations in the economy.

(c) [7 marks]

Suppose each agent has an endowment of half-unit of each good. Prove **without direct computation** that the competitive equilibrium allocation is both Pareto-efficient and envy-free.

3. [25 marks: $7+3+3+12$]

Two flat-mates, 1 and 2, rent a flat and play their own music on the only CD player owned by the flat-owner. They both like their own music, but dislike the music played by the other person. Given the timing constraints, each one must play her own music when the other person is also present. Let m_i denote the amount of music played by i, and Y_i denote her amount of money holding. Individual i's utility function is

$$u_i(m_1, m_2, Y_i) = 8m_i - 2m_i^2 - \frac{3}{2}m_j^2 + Y_i, \;\; i, j = 1, 2, \; i \neq j.$$

(a) [7 marks]

How much music would each individual play? What is the efficient amount of music for each individual? Is the amount of music actually played more or less than the efficient level? Explain the economic intuition for your answer.

(b) [3 marks]

Suppose that individual 2 is considering to gift a headphone to her flat-mate on her birthday. Assume that she does not get any utility from just gift-giving. What is the maximum price she is willing to pay for the headphone?

(c) [3 marks]

Suppose that the price of the headphone is Rs. 11. Does it make sense for the two flat-mates to jointly buy a headphone, sharing the price equally, and making a binding commitment that they would each listen to their own music only via the headphone?

(d) [12 marks]

Now suppose that the CD player is owned by individual 1 so that she can prevent individual 2 from playing any music at all. Suppose individual 1 can offer a *take-it-or-leave-it* contract that looks like the following:

"I shall play music at a level $\bar{m}_1$, and you can play music at the level $\bar{m}_2$ in return for a sum of Rs. T."

In case the offered contract is rejected, individual 1 selects m_1 unilaterally, and individual 2 cannot play any music of her choice. Solve for the optimal levels of $\bar{m}_1$, $\bar{m}_2$ and T. Discuss the economic intuition for your answer.

Group B

1. [25 marks: $2 + 10 + 13$]

   Consider a country where there are only two provinces – A and B. The production function to produce a single output Y is given by $Y = F(N^A + N^B)$ where F is a concave function and N^i represents employees from province i, $i = A, B$. Wages paid to the employees are given by W^i, $i = A, B$. Price of the final good Y is denoted by P. The employers are price takers and take P, W^A and W^B as given.

   (a) [2 marks]

   Write down the expression for an employer's profit as a function of N^A and N^B, $\pi\left(N^A, N^B\right)$.

   (b) [10 marks]

   An employer chooses N^A and N^B to maximize

   $$u\left(N^A, N^B\right) = u\left(\pi\left(N^A, N^B\right), N^A, N^B\right),$$

   where $\frac{\partial u}{\partial \pi} > 0$, $\frac{\partial u}{\partial N^A} > 0$ and $\frac{\partial u}{\partial N^B} < 0$. The last two conditions on $u\left(N^A, N^B\right)$ imply that the employer prefers employees from province A but dislikes employees from province B.

   Write down the first order conditions for the employer's maximization problem assuming an interior solution.

   (c) [13 marks]

   In equilibrium do the employees from different provinces get the same wage? If yes, explain your answer. If not, then determine, with a clear explanation, which employees are paid more and by how much.

2. [25 marks: $4 + 5 + 8 + 8$]

Consider a concave utility function $u(c, l)$ where c represents consumption good and l represents labour supply (working hours, to be precise). While utility increases with the level of consumption good, increasing working hours reduces utility. Wage per hour of labour is given by w, thus working for l hours will ensure wl amount of total wage which is denoted by y, that is, $y = wl$. Given this, the utility function can be written as $u(c, \frac{y}{w})$. The price of the consumption good c is given by p. Also $\overline{L}$ is a fixed number of hours representing total time available to an agent and $\overline{L} - l$ represents leisure. [In all the figures you are asked to draw below, plot y on x-axis and c on y-axis.]

(a) [4 marks]

Derive the slope of an indifference curve for the utility function $u(c, \frac{y}{w})$ on the y-c plane.

(b) [5 marks]

Demonstrate the agent's utility maximizing choice of y and c in a figure by plotting her budget line and indifference curves for the utility function $u(c, \frac{y}{w})$.

(c) [8 marks]

Experiment 1: Suppose there is an increase in w. Demonstrate the agent's new utility maximizing choice of y and c in the same figure as in part (b). [Show clearly how the agent's budget line and/or indifference curves change as a result of the increase in w.] Compare the old and new choices with a brief economic explanation.

(d) [8 marks]

Experiment 2: Suppose, instead of an increase in w, there is a tax imposed on income. That is, the after-tax income of the agent is $(1-\tau)\,y$ where τ is the proportional tax rate. In a new figure demonstrate the agent's new as well as old (as in part (b)) utility maximizing choices of y and c. [Show clearly how the agent's budget line and/or indifference curves change as a result of this proportional tax.] Compare the old and new choices with a brief economic explanation.

3. [25 marks: $4+3+10+4+4$]

Consider an agent who lives for three periods but consumes only in periods two and three where the consumptions are denoted by c_2 and c_3 respectively. Her utility is given by $u\,(c_2, c_3) = \log(c_2) + \beta\log(c_3)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. She invests an amount e in education in the first period which she borrows from the market at a given interest rate $r > 0$. Her income in the second period is $w \cdot h(e)$ where w is a fixed wage rate per unit of human capital and $h(e)$ is the amount of human capital that results from investment in education (e) in the first period. Assume that $h(e)$ is an increasing and concave function of e. The agent repays her education loan in the second period. She has no income in the third period. But she can save (s) in the second period from her income on which she receives the return $s\,(1+r)$ in the third period to meet her consumption expenditure.

(a) [4 marks]

Write down the agent's period 2 and period 3 budget constraints separately.

(b) [3 marks]

Set up the agent's utility maximization problem by showing her choice variables clearly.

(c) [10 marks]

Write down the first order conditions for the agent's utility maximization problem.

(d) [4 marks]

Derive the ratio of consumptions in period 2 and period 3, $\frac{c_2}{c_3}$, in terms of the parameters of the model.

(e) [4 marks]

Explain how investment in education, e, depends on the preference parameter β.

Group C

1. [25 marks: $5 + 10 + 10$]

   Consider a street represented by the interval $[0, 1]$. Three agents, $\{1, 2, 3\}$, live on this street. Agent $i \in \{1, 2, 3\}$ lives at $x_i \in [0, 1]$, and assume that $x_1 \leq x_2 \leq x_3$. Suppose we locate a hospital at a point $p \in [0, 1]$.

   (a) [5 marks]

   We say p is **square-optimal** if it minimizes $\sum_{i=1}^{3}(x_i - p)^2$. Derive the square optimal value of p.

   (b) [10 marks]

   We say p is **absolute-optimal** if it minimizes $\sum_{i=1}^{3} |x_i - p|$.

   i. Argue that if p is absolute-optimal, then $p \in [x_1, x_3]$.

   ii. Use this to derive an absolute-optimal p.

   (c) [10 marks]

   Now suppose that n agents, $\{1, 2, 3, ..., n\}$, live on this street where $x_1 \leq x_2 \leq x_3 ... \leq x_n$ and n is an odd number. Derive an absolute-optimal p.

2. [25 marks: $7 + 5 + 5 + 8$]

   Two random variables x_1 and x_2 are uniformly drawn from $[0, 1]$. Define the following function:

$$G(p) = p \times \text{Probability}[\ p \geq \max(x_1, x_2)] \qquad \forall\ p \in [0, 1].$$

   (a) [7 marks]

   Derive, with a clear explanation, the expression for $G(p)$.

(b) [5 marks]

Plot $G(p)$.

(c) [5 marks]

Is G convex or concave in p? Give clear explanations for your answer.

(d) [8 marks]

Find $\max_{p\in[0,1]} G(p)$.

3. [25 marks: $7+9+9$]

Let $X \subset \mathbb{R}$ and $f : X \to X$ be a continuous function.

(a) [7 marks]

Suppose $X = [0,1]$. By using the Intermediate Value Theorem, show that there exists $x^* \in X$ such that $f(x^*) = x^*$.

(b) [9 marks]

In each of the cases below, determine whether there exists $x^* \in X$ such that $f(x^*) = x^*$. Justify your claim by either providing a proof or a counter-example.

i. $X = (0,1)$ and f is continuous.

ii. $X = [0,1] \cup [2,3]$ and f is continuous.

iii. $X = [0,1]$ but f is not continuous.

(c) [9 marks]

Let $f_i : [0,1] \to [0,1]$, $i = 1,2,\ldots,m$, be a collection of m continuous functions. Prove that there exists $x^* \in [0,1]$ such that $\sum_{i=1}^{m} f_i(x^*) = mx^*$.

ISI 2020 PEA

1. Consider the functions

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

(a) f is differentiable at zero but g is not differentiable at zero

(b) g is differentiable at zero but f is not differentiable at zero

(c) f and g are both differentiable at zero

(d) Neither f nor g is differentiable at zero

2. How many ordered pairs of numbers (x, y) are there, where $x, y \in \{1, 2, \ldots, 100\}$, such that $|x - y| \leq 50$?

(a) 2550

(b) 5050

(c) 7550

(d) None of the other options are correct

3. Let ABC be a right angled isosceles triangle with angle $\angle ABC$ being right-angled. Let D be the mid-point of AB, E be the foot of the perpendicular drawn from D to the side AC, and F be the foot of the perpendicular drawn from E to the side BC. What is the value of $\frac{FC}{BC}$?

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{3}{4}$

(c) $2 - \sqrt{2}$

(d) None of the other options are correct

4. Suppose that there are 30 MCQ type questions where each question has four options: A, B, C, D. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, and 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5, option B is correct is 0.3, option C is correct is 0.2, and option D is correct is 0. Two students Gupi and Bagha have no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5, but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

(a) Both Gupi and Bagha have expected scores more than 30

(b) Gupi's expected score is greater than or equal to 30 and Bagha's expected score is strictly less than 30

(c) Gupi's expected score is less than or equal to 30 and Bagha's expected score is strictly more than 30

(d) None of the other options are correct

5. Evaluate: $\lim_{x\to\infty}\left[e^{3x}-5x\right]^{\frac{1}{x}}$

(a) e^3

(b) 3

(c) 1

(d) None of the other options are correct

6. Suppose $f(x)=\left\{\begin{array}{ll}\frac{|x-3|}{x-3}, & \text{for } x\neq 3\\ 0, & \text{for } x=3\end{array}\right\}$. Then, $\lim_{x\to 3} f(x)$:

(a) is -1

(b) is 0

(c) does not exist

(d) is 1

7. Consider the following system of equations in x, y, z :

$$\begin{aligned} x+2y-3z &= a\\ 2x+6y-11z &= b\\ x-2y+7z &= c \end{aligned}$$

For what values of a, b, c, does the above system have no solution?

(a) $c+2b-5a\neq 0$

(b) $c+2b-5a=0$

(c) $c+2b-4a=0$

(d) None of the other options are correct

8. The sequence x_n is given by the formula: for every positive integer n

$$x_n=n^3-9n^2+631.$$

The largest value of n such that $x_n > x_{n+1}$

(a) is 4
(b) is 5
(c) is 6
(d) None of the other options are correct

9. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D?

(a) 10
(b) 1
(c) 0
(d) None of the other options are correct

10. Suppose we are given a 4×4 square matrix A, which satisfies $A_{ij} = 0$ if $i < j$. Suppose the each diagonal entry A_{ii} is drawn uniformly at random from $\{0, 1, \ldots, 9\}$. What is the probability that A has full rank?

(a) $\frac{1}{10^4}$
(b) $\frac{3}{5}$
(c) $1 - \frac{1}{10^4}$
(d) $\left(\frac{9}{10}\right)^4$

11. Let a and b be two real numbers where $b \neq 0$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function which satisfies

$$g(g(x)) = ag(x) + bx \qquad \forall\, x \in \mathbb{R}.$$

Which of the following must be true?

(a) g is strictly increasing
(b) g is strictly decreasing
(c) $\lim_{x\to\infty} g(x)$ is finite
(d) Either (a) or (b)

12. For every positive integer n, let $S(n)$ denote the sum of digits in n. For instance, $S(387) = 3 + 8 + 7 = 18$. The value of the sum

$$S(1) + S(2) + \ldots + S(99)$$

is

(a) 450
(b) 495
(c) 900
(d) 990

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 playing cards, with four suits of 13 cards each, which has been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

(a) $\frac{{}^4C_1\ {}^{13}C_5}{{}^{52}C_5}$
(b) $\frac{{}^4C_2\ {}^{13}C_4}{{}^{52}C_5}$
(c) $\frac{{}^4P_1\ {}^{13}C_5}{{}^{52}P_5}$
(d) $\frac{{}^4C_1\ {}^{12}C_5}{{}^{52}C_5}$

14. Evaluate: $\int x^n \ln x dx$, where $n > 1$

(a) $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)^2} x^{n+1} + c$
(b) $\ln x \frac{x^{n+1}}{2(n+1)} - \frac{1}{(n+1)^2} x^{n+1} + c$
(c) $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)} x^{n+1} + c$
(d) None of the other options are correct

15. The area of the region bounded by the curve $y = \ln(x)$, the $Y - axis$, and the lines $y = 1$ and $y = -1$

(a) is $\frac{e}{2}$
(b) is 2
(c) is $e - \frac{1}{e}$
(d) None of the other options are correct

16. A price discriminating monopolist finds that a person's demand for its product depends on the person's age. The inverse demand function of someone of age y, can be written $p = A(y) - q$ where $A(y)$ is an increasing function of y. The product cannot be resold from one buyer to another and the monopolist knows the ages of its consumers. (This is often the case with online subscriptions.) If the monopolist maximizes its profits, then

(a) older people will pay higher prices and purchase more of this product compared to younger people

(b) everyone pays the same price but older people consume more

(c) older people will pay higher prices compared to the younger people but everyone will consume the same quantity of the product

(d) None of the other options are correct

17. Pam's family consists of herself and her 3 sisters. They own a small farm in the agricultural sector in Agri-land. The value of their total output is $4000 which is divided equally amongst the four. The urban sector has two kinds of jobs: informal sector (which anyone can get) pays $500 and formal sector jobs give $1200. Pam would like to maximize her own total income and calculates her own expected returns to migration. The proportion of formal sector jobs to urban labor force that would deter her from migrating is:

    (a) Less than 2/3

    (b) More than 5/6

    (c) More than $\frac{1}{2}$

    (d) Less than 5/7

18. Hu and Li are two dealers of used tractors in a rural area of China. Hu sells high quality second hand tractors while Li sells low quality ones. Hu would be willing to sell his high quality tractor at $8000 while Lu would sell his low quality one for $5000. Consumers are willing to pay up to $10,000 for a high quality tractor and $7000 for a low quality one. They expect a 50% chance of buying a high quality second-hand tractor. In order to signal the quality of their tractors Hu and Li can offer warranties. The cost of warranty for a high quality tractor is 500Y and 1000Y for a low quality one (Y is the number of years of warranty). What is the optimal number of years of warranty that Hu should offer so that consumers know his tractors are of good quality?

    (a) Less than 2 years

    (b) 0.5 years

    (c) More than 1.5 years

    (d) 3 years

19. Suppose the capacity curve for each laborer is described as follows: for all payments up to $100, capacity is zero and then begins to rise by 2

units for each additional \$ paid. This happens until the payment rises to \$500. Thereafter, an additional \$ payment increases work capacity by only 1.1 units, until total income paid is \$1000. At this point additional payments have no effect on work capacity. Assume all income is spent on nutrition. Suppose you are an employer faced by the above capacity curve of your workers. You need 8000 units of work or capacity units. How many workers would you hire and how much would you pay each worker so that you get 8000 units of work at minimum cost?

(a) 5 workers; \$1000 per worker

(b) 10 workers; \$700 per worker

(c) 10 workers; \$500 per worker

(d) 15 workers; \$400 per worker

20. Suppose you were to believe that "money illusion" exists, that is as prices and income rise proportionally, then people buy more. Which of the following statements about demand should not be true?

(a) Demand functions are downward sloping

(b) Demand functions are homogenous of degree zero

(c) Demand has a positive vertical intercept

(d) Demand functions are homogenous of degree one

21. Consider a Bertrand price competition model between two profit maximizing widget producers, say A and B. The marginal cost of producing a widget is 4 for each producer. Each widget producer has a capacity constraint to produce only 5 widgets. There are 8 identical individuals who demand 1 widget only, and value each widget at 6. If the firms are maximizing profits, then the following statement is true:

(a) Firm A and B will charge 4

(b) Firm A and B will charge 6

(c) Firm A and B will charge greater than or equal to 5

(d) None of the other options are correct

22. The government estimates the market demand (Q_D) and market supply (Q_S) for turnips to be the following: $Q_D = 30 - 2P$, $Q_S = 4$; where P is the per unit price and Q is the quantity measured in kilograms. The government aims to increase the market price of turnips to \$8 per unit

to improve the welfare of domestic producers of turnips. Is is considering three possible choices: (i) a per unit subsidy; (ii) a price floor and purchase of any surplus production, and (iii) a production quota. Which of these policies should the government adopt if it wants to maximize the producers' welfare but minimize the loss of efficiency?

(a) A production quota

(b) A price subsidy

(c) Either a price subsidy or a price floor

(d) Either a production quota or a price floor

23. A monopolist faces a demand curve: $q = \frac{5}{p}$. Her cost function is: $C(q) = 3q$. Suppose, in the same market, there are some competitive suppliers ready to sell the good at the price $p = 5$. The monopolist's profit maximizing price and output could be given by

(a) $p = 3$, $q = \frac{5}{3}$.

(b) $p = 3.01$, $q = \frac{5}{3.01}$

(c) $p = 2.99$, $q = \frac{5}{2.99}$

(d) $p = 4.99$, $q = \frac{5}{4.99}$

24. The consumption function is given by $C = AY^{\beta}$ with $\beta = 0.5$ and $A = 0.3$. The marginal propensity to save is

(a) equal to 0.5

(b) increasing in income, Y

(c) equal to 0.3

(d) equal to 0.7

25. The production function is given by $Y = AL$. The wage rigidity constraint is given by $W \geq B$. The labour endowment is given by C. Here A, B, and C are finite and positive constants. Assume that the entire labor endowment is supplied. If $A > B$, then in a labour market equilibrium

(a) $L = C$

(b) $L = 0$

(c) $0 < L < C$

(d) None of the other options are correct

26. Consider the Mundell-Flemming model with perfect capital mobility and a flexible exchange rate in the short run. A monetary expansion leads to a/an _______ in output; a fiscal expansion leads to _____ in output.

(a) decrease; no change

(b) increase; decrease

(c) increase; no change

(d) increase; increase

27. Mr. X has an exogenous income W and his utility from consumption c is $u(c)$. Mr. X knows that an accident can occur with probability p and if it occurs, the monetary equivalent to the damage is T. Mr. X can however affect the accident probability p through the prevention effort e. In particular, e can take two values - zero and a and an assumption is that $p(0) > p(a)$, that is by putting prevention effort, probability of occurring an accident can be reduced. Let us also assume that if Mr. X puts an effort e, the disutility from the effort is Ae^2 where A is the per unit effort cost. What is the critical value of A, A^*, below which the effort will be undertaken, and above which the effort will not be undertaken, by Mr. X ?

(a) $A^* = \frac{[p(a)-p(0)][u(W-T)-u(W)]}{a^2}$

(b) $A^* = \frac{[p(a)-p(0)]a^2}{[u(W-T)-u(W)]}$

(c) $A^* = \frac{\frac{p(a)}{p(0)}}{\frac{u(W-T)}{u(W)}a^2}$

(d) $A^* = \frac{p(a)p(0)a^2}{u(W-T)u(W)}$

28. Labor supply in macro models results from individual decision making. Let c denote an individual's consumption and L denote labor supply. Assume that individuals solve the following optimization problem

$$\underset{\{c,L\}}{Max}\, U(c,L) = \log c - \frac{1}{2}\frac{1}{b}L^2$$

subject to $c + \bar{S} = wL$ where $U(\cdot)$ is the utility function, $b > 0$ is a constant, $\bar{S}$ is a constant exogenous level of savings, and w is the real wage the person can earn in the labor market. Derive the optimal labor supply. It is

(a) increasing in w; increasing in c

(b) decreasing in w; decreasing in c

(c) increasing in w; decreasing in c

(d) decreasing in w; increasing in c

29. Consider a Solow economy that begins in steady state. Then a strong earthquake destroys half the capital stock. The steady state level of capital______, the level of output_____on impact, and the growth rate of the economy_____as the economy approaches its steady state.

(a) decreases; decreases; decreases

(b) remains the same; decreases; decreases

(c) remains the same; decreases; increases

(d) decreases; remains the same, decreases

30. Suppose the economy is characterized by the following equations

$$\begin{aligned} C &= c_0 + c_1 Y_D \\ Y_D &= Y - T \\ I &= b_0 + b_1 Y \end{aligned}$$

where C = Consumption, c_0 = Autonomous Consumption, $c_1 \in [0,1]$, Y_D = Disposable Income, Y = Aggregate GDP, T = Taxes, I = Investment, b_0 = Autonomous Investment, and $b_1 \in [0,1]$. For the multiplier to be positive, what condition needs to be satisfied?

(a) $b_1 + c_1 = 0$

(b) $b_1 + c_1 = 1$

(c) $b_1 + c_1 < 1$

(d) $b_1 + c_1 > 1$

Group A

1. [30 marks: 5+10 +15].

Let $X_1, ..., X_n$ be independent and identically distributed random variables with a uniform distribution on $(0, \theta]$ where $\theta > 0$.

(a) **[5 marks]** Write down the joint probability density function of $X_1, ..., X_n$.

(b) **[10 marks]** Suppose x_i is a realization of X_i, for each $i = 1, ..., n$. And suppose the value of θ is unknown. Find the value of θ that maximizes the joint p.d.f. in part (a) *given* that $x_1, ..., x_n$ have been observed. (This is called the maximum likelihood estimate of θ.)

(c) **[15 marks]** Consider the function: $f(x, y) = x^2 + y^2 - 2x$

(i) Find the maximum value of f over the region $\{(x, y) \mid 2x^2 + 3y^2 - 2x \leq 100\}$

(ii) Find the minimum value of f over the region $\{(x, y) \mid 2x^2 + 3y^2 - 2x \geq 100\}$

2. [30 marks: 3+5+10+6+6]

A *tournament* consists of n players and all possible $C(n, 2) = \frac{n(n-1)}{2}$ pairwise matches between them. There are no ties in a match: in any match, one of the two players wins. The score of a player is the number of matches she wins out of all her $(n-1)$ matches in the tournament. Denote the score vector of the tournament as $s \equiv (s_1, \ldots, s_n)$ and assume without loss of generality $s_1 \geq s_2 \geq \ldots \geq s_n$.

(a) **(3 marks)** For any $2 \leq k \leq n$, show that $s_1 + \ldots + s_k \geq C(k, 2)$, where $C(k, 2) = \frac{k(k-1)}{2}$.

(b) **(5 marks)** Suppose $n > 3$ and players $1, 2, 3$ win every match against players in $\{4, \ldots, n\}$. Find the value of $s_4 + \ldots + s_n$?

(c) **(10 marks)** Suppose $s_n = s_0, s_{n-1} = s_0 + 1, s_{n-2} = s_0 + 2$ for some positive integer s_0 and $n \geq 3$. Show that

$$s_0 \leq \frac{(n-2)(n-3)}{2n}.$$

(d) **(6 marks)** A tournament generates a score vector s such that

$$s_j - s_{j+1} = 1 \text{ for all } \ j \in \{1, \ldots, n-1\}.$$

What is the score vector of this tournament? For every Player j, who does Player j beat in this tournament?

(e) (6 marks) Suppose there are six players, i.e., $n = 6$. There is a tournament such that each player has a score of at least two and difference in scores of any two players is not more than one. What is the score vector of this tournament? Construct a tournament (describing who beats who) which generates this score vector.

3. [30 marks: 6+6+6+4+8]

Consider the following equation in x:

$$(x-1)(x-2)\cdots(x-n) = k, \tag{1}$$

where $n > 1$ is a positive integer and k is a real number. Argue whether the following statements are true or false by providing a proof or a counter example.

(a) (6 marks) Suppose $n = 2$. There is a real solution to Equation (1) for every value of k.

(b) (6 marks) Suppose $n = 3$. There is a real solution to Equation (1) for every value of k.

(c) (6 marks) For all $k \geq 0$ and for every positive integer $n > 1$, there is a real solution to Equation (1).

(d) (4 marks) For all $k < 0$ and for every *odd* positive integer $n > 1$, there is a real solution to Equation (1)

(e) (8 marks) For all $k < 0$, there is *some even* positive integer n such that a real solution to Equation (1) exists.

ISI 2020 PEB

Group B

1. Consider an economy inhabited by identical agents of size 1. A representative agent's preference over consumption (c) and labour supply (l) is given by the utility function

$$u(c,l) = c^{\alpha}(24-l)^{1-\alpha}, \; 0 < \alpha < 1.$$

Production of the consumption good c is given by the production function $c = Al$, where $A > 0$ is the productivity of labour. Both the commodity market and labour market are perfectly competitive: the buyers and sellers take the price as given while taking demand and supply decisions. Let us denote the hourly wage rate by $w > 0$ and price of the consumption good by $p > 0$.

(a) **[25 marks: 13 + 7 + 5]** Competitive Equilibrium:

A competitive equilibrium is given by the allocation of consumption and labour, (c^{CE}, l^{CE}), and the relative price ratio, $\frac{w}{p}$, such that, given w and p, a representative agent decides her labour supply, l^S, and consumption demand, c^D, to maximize her utility; a firm decides its labour demand, l^D, and supply of consumption good, c^S, to maximize its profit; and, finally, both the commodity market and labour market clear, that is, $l^D = l^S$ and $c^D = c^S$.

(i) [**13 marks**] Set up the representative agent's utility maximization problem. Write down the first order conditions for this maximization problem and determine l^S and c^D as functions of w and p.

(ii) [**7 marks**] Set up a firm's profit maximization problem. Determine l^D and c^S as functions of w and p.

(iii) [**5 marks**] Determine the competitive equilibrium allocation, (c^{CE}, l^{CE}), and the relative price ratio, $\frac{w}{p}$.

(b) **[5 marks]** Pareto efficient allocation:

For this economy define the concept of a Pareto efficient allocation of consumption and labour. Find out a Pareto efficient allocation of consumption and labour in this economy. Provide a clear explanation.

2. [30 marks = 12 + 18]

(a) [**12 marks**] Ms. A's income consists of Rs.1,00,000 per year from pension plus the earnings from whatever she sells of the 2,000 kilograms of rice she harvests annually from her farm. She spends this income on rice (x) and on all other expenses (y). All other expenses (y) are measured in rupees, so that the price of y is Rs. 1. Last year rice was sold for Rs. 20 per kilogram, and Ms. A's rice consumption was 2,000 kilograms, just the amount produced on her farm. This year the price of rice is Rs. 30 per kilogram. Ms. A has standard convex preferences over rice and all other expenses. *Answer the following two questions without referring to any utility function or indifference curves.*

(i) [**7 marks**] What will happen to her rice consumption this year – increase, decrease, or remain the same? Give a clear explanation for your answer.

(ii) [**5 marks**] Will she be better or worse off this year compared to last year? Explain clearly.

(b) [**18 marks**] There are two goods x and y. Mr. B has standard convex preferences over the two goods. He has endowments of $e_x > 0$ units of good x and $e_y > 0$ units of good y. He does not have any other source of income. When the price of good y is Rs. 1 and the price of good x is Rs. p_x, he decides neither to buy nor to sell good x.

(i) [**8 marks**] Suppose that, for good x, the prices have become Rs. $p_L < p_x$ if an individual is a *seller* and Rs. $p_H > p_x$ if an individual is a *buyer*. The price of good y remains Rs. 1 no matter whether an individual buys or sells good y. Write down the equation of the new budget constraint and draw it labelling the important points clearly.

(ii) [**10 marks**] Will Mr. B buy or sell good x? By how much? Give a clear explanation for your answer without referring to any utility function or indifference curves.

3. [30 marks = 6+7+7+10]

Consider a moneylender who faces two types of potential borrowers: the *safe type* and the *risky type*. Each type of borrower needs a loan of the same size L to invest in some project. The borrower can repay only if the investment produces sufficient returns to cover the repayment. Suppose that the safe type is always able to obtain a secure return of R from the investment, where $R > L$. On the other hand, the risky type is an uncertain prospect; he can obtain a higher return R' (where $R' > R$), but only with probability p. With probability $1-p$, his investment backfires and he gets a return of 0. The money lender has enough funds to lend to just one applicant, and there are two of them – one risky, one safe. Each borrower knows his own type, but the moneylender does not know the borrower's type. He just knows that one borrower is a safe type and the other one is a risky type. Since the moneylender has enough funds to lend to just one applicant, when both the borrowers apply for the loan, he gives the loan randomly to one of them, say by tossing a coin. Assume that the lender supplies the loan from his own resources and his opportunity cost is zero.

(a) **[6 marks]** What is the highest interest rate, call it i_s, for which the safe borrower wants the loan? What is the highest interest rate, i_r, for which the risky borrower wants the loan? Who is willing to pay a higher interest rate, the risky borrower or the safe borrower?

(b) **[7 marks]** The lender's objective is to maximize his expected profit. Argue clearly that the lender's effective choice is between two interest rates, i_s and i_r. (That is, argue that the lender will *not* choose any interest rate strictly lower than min $\{i_r, i_s\}$, any interest rate strictly higher than max $\{i_r, i_s\}$, or any interest rate strictly in between i_r and i_s.)

(c) **[7 marks]** Argue that when the lender charges i_r, his expected profit is given by $p(1+i_r)L - L$. Derive, with a clear argument, the expression of lender's expected profit when he charges i_s.

(d) **[10 marks]** An equilibrium with *credit rationing* occurs when, at the equilibrium interest rate, some borrowers who want to obtain loans are unable to do so; however, lenders do not raise the interest rate to eliminate the excess demand.

Explain clearly that we have an equilibrium with *credit rationing* when

$$p < \frac{R}{2R' - R}.$$

Group C

1. [30 marks=3+8+2+10+4+3]

Consider an economy where identical agents (of mass 1) live for two periods: youth (period 1) and old age (period 2). The utility function of a representative agent born at time t is given by

$$u\left(c_{1,t}, c_{2,t+1}\right) = \log\left(c_{1,t}\right) + \beta \log\left(c_{2,t+1}\right),$$

where c_1 denotes consumption in youth, c_2 denotes consumption in old age, and $0 < \beta < 1$ is the discount factor reflecting her time preference. In her youth the representative agent supplies her endowment of 1 unit of labour inelastically and receives the market-determined wage rate w_t. So in her youth the agent faces the budget constraint $c_{1,t} + s_t = w_t$, where s_t denotes her savings. When old, she just consumes her savings from youth plus the interest earning on her savings, $s_t r_{t+1}$, where r_{t+1} is the market-determined interest rate in period $t+1$. That is, when old, her budget constraint is $c_{2,t+1} = \left(1 + r_{t+1}\right) s_t$.

(a) [**3 marks**] Set up the agent's utility maximization problem by showing her choice variables clearly.

(b) [**8 marks**] Write down the first order conditions for this maximization problem and derive the savings function. Explain how savings, s_t, if it does, depends on the interest rate r_{t+1}.

The production function of the economy is given by $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, $0 < \alpha < 1$, where K and L denote the amounts of capital and labour in the economy, respectively. Capital depreciates fully after use, that is, the rate of depreciation of capital is one. Factor markets being competitive, the equilibrium factor prices are given by their respective marginal products.

(c) [**2 marks**] Derive the equilibrium wage rate (w_t) of the economy in terms of K_t. [Keep in mind that the mass of agents is 1 and each agent supplies her endowment of 1 unit of labour inelastically.]

The role of the financial sector (banks, stock market, and so on) is to mobilize the savings of households to bring it for effective use by the production sector. But the financial sector does not work well and a fraction $0 < \theta < 1$ of aggregate savings gets lost (vanishes in thin air) in the process of intermediation.

(d) [**10 marks**] Derive the law of motion of capital (that is, express capital in period $t+1$, K_{t+1}, in terms of capital in period t, K_t).

(e) [**4 marks**] Derive the steady state amount of capital of the economy.

(f) [**3 marks**] How does the steady state amount of capital depend on the inefficiency of the financial sector θ?

2. [30 marks = 5+5+10+10]

Consider a Solow-Swan model with learning by doing. Assume that the production function is of the form

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

where A is the level of technological progress and grows at the rate $g > 0$, L is the population with grows at the rate $n > 0$, K is the capital stock, Y is GDP, and $\alpha \in [0, 1]$. Assume that

$$\dot{K} = sY - \delta K$$

Define $Z = \frac{K}{AL}$ as the capital labor ratio in efficiency units. Let output per worker be given by $Q = AZ^{\alpha}$. The parameter $s \in [0, 1]$ denotes the savings rate. The parameter $\delta \in [0, 1]$ denotes the depreciation on capital.

(a) [**5 marks**] Derive an expression for $\frac{\dot{Z}}{Z}$

(b) [**5 marks**] Instead of assuming that the rate of technological progress is constant (g), now assume that the instantaneous increase in A is proportional to output per worker, i.e., there is learning by doing

$$\dot{A} = \gamma Q.$$

Show that the law of motion of capital is given by

$$\dot{Z} = (s - \gamma Z)Z^{\alpha} - (\delta + n)Z$$

(c) [**10 marks**] Draw a diagram describing the dynamics of growth in the model with learning by doing. Plot Z on the $x - axis$, and the appropriate functions on the $y - axis$

(d) [**10 marks**] In contrast to the model with no learning by doing, does an increase in the investment rate raise the balanced-growth rate ? What does this tell you about the change in policy having level effects versus growth effects in the model with learning by doing in contrast to the model when there is no learning by doing ? Show your answer using the diagram in part (c).

3. [30 marks =10+10+3+3+4]

Suppose households who live till T periods maximize $\sum_{n=t}^{T} \beta^{n-t} \ln(c_n)$ where c_n represents their income in period $n = t, t+1, t+2, ...T$ and β is a parameter with $0 < \beta < 1$. Suppose per period income and the saving of households are y_n and s_n respectively and the activity starts from the beginning of their life t. Further, the net interest rate on saving in between any two periods is exogenously fixed at r and so the gross rate of return is $1 + r$. Households have only two activities in every period - consuming and saving.

(a) [**10 marks**] Write down the sequence of budget constraints (one for each period) and the aggregate budget constraint derived from these periodic budget constraints where on the left hand side, consumption levels for all the periods appear, and on the right hand side, income in all periods appears.

(b) [**10 marks**] Under what condition between β and r, is the optimal solution for the above problem yield constant consumption, $\widetilde{C}$, in every period ?

(c) Suppose the condition that you derive in (b) holds. Then answer the following questions in (c) and (d)

(i) [**3 marks**] For a transitory change in income in period t only, calculate the change in the constant level of consumption, $\widetilde{C}$.

(ii) [**3 marks**] For a transitory change in income in period $t + k$ only, calculate the change in the constant level of consumption, $\widetilde{C}$.

(d) [**4 marks**] For a permanent change in income (assume the same amount of income change in all periods), calculate the change in the constant level of consumption, $\widetilde{C}$. Compare this value derived with part c (i).

ISI 2021 PEA

1. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n^2+2}{3n^2+1}$, then $\sum_{n=1}^{\infty} a_n$ is
   (A) 0
   (B) divergent
   (C) $\frac{2}{3}$
   (D) $\frac{3}{2}$

2. Using data from a sample of size n, the intercept and slope coefficients from an ordinary least squares regression of y on x, are a and b respectively. Which of the following is **false**?
   (A) $\sum_{i=1}^{n}(y_i - a - bx_i)x_i = 0$
   (B) $\frac{1}{n}\sum_{i=1}^{n} y_i = a + \frac{b}{n}\sum_{i=1}^{n} x_i$
   (C) a and b are the solution to $\min_{\alpha,\beta}\sum_{i=1}^{n}(y_i - \alpha - \beta x_i)^2 = 0$
   (D) a and b are the solution to $\min_{\alpha,\beta}\sum_{i=1}^{n}|y_i - \alpha - \beta x_i| = 0$

3. Consider a production function $z = 2x + 3y$. For what price ratio $\frac{p_x}{p_y}$, will a corner solution in y, i.e. $(x = 0)$ be possible, if the objective is to minimize the cost of producing a given positive quantity z_0 of z?
   (A) $\frac{p_x}{p_y} = 2/3$
   (B) $\frac{p_x}{p_y} \geq 2/3$
   (C) $\frac{p_x}{p_y} < 2/3$
   (D) $\frac{p_x}{p_y} \leq -2/3$

4. A number is chosen randomly from the first billion natural numbers. The probability that the product of the number with its two immediate successors is divisible by 24 is closest to
   (A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$

5. Each of the four entries of a 2×2 matrix is filled by independently choosing either 1 or -1 uniformly at random. What is the probablity that the matrix is singular?
   (A) $\frac{1}{16}$
   (B) $\frac{1}{4}$
   (C) $\frac{1}{2}$
   (D) $\frac{1}{3}$

6. We need to fill a 3×3 matrix by either 0 or 1 such that each row has exactly one 0 and each column has exactly one 0. The number of ways we can do this is

(A) 8 (B) 6

(C) 4 (D) 2

7. Let $f : [0, 1] \to \mathbb{R}$ be a convex function with $f(0) = 0$. Which of the following is always true for f?

(A) f is differentiable

(B) f may not be differentiable but it is continuous

(C) $f(x) \geq xf'(x)$ for all $x \in [0, 1]$ if f is differentiable

(D) none of the above

8. If $f : \mathbb{R} \to \mathbb{R}$ is strictly quasi-concave, then it follows that

(A) f is not strictly convex

(B) f is not linear

(C) f is monotonic

(D) if f is quadratic, then the coefficient of $x^2 \leq 0$

9. The rank of the matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

is

(A) 0 (B) 1

(C) 2 (D) 3

10. Let V be the vector space of polynomials $p(x)$ of degree less than or equal to 2 that have real coefficients. Then T is a linear transformation from V to V if T is defined by

(A) $T(p(x)) = x + p(x)$

(B) $T(p(x)) = xp(x)$

(C) $T(p(x)) = \frac{dp(x)}{dx}$

(D) $T(p(x)) = \int p(x)dx$ where the constant of integration is taken to be zero.

11. X is a random variable that can take values only in $[0, 10]$. $P(X > 5) \leq \frac{2}{5}$ and $P(X < 1) \leq \frac{1}{2}$. Then

(A) $E(X) \geq 1$

(B) $E(X) \leq 5$

(C) $E(X) \geq 0.5$ and $E(X) \leq 8.5$

(D) None of the above is true

12. Given data $(-1, 1), (0, 0), (1, 1)$ on (x, y), the standard deviations of x and y and the correlation coefficient of x and y are respectively

(A) $\sigma_x = \sqrt{2}/\sqrt{3}, \quad \sigma_y = \sqrt{2}/3, \quad r = 0$

(B) $\sigma_x = 2/3, \quad \sigma_y = 2/9, \quad r = 0$

(C) $\sigma_x = 0, \quad \sigma_y = 0, \quad r$ is undefined

(D) None of the above

13. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are f_1 and f_2 respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn unformly from

$[0,1]$. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from $[0,1]$.

Total demand of vaccine is 1 unit and if firm $i \in \{1,2\}$ supplies $q_i \in [0,1]$ units and has a per unit cost of c_i, it incurs a manufacuring cost of $c_i q_i$ (along with f_i).

Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

(A) $f_1 + f_2 + \frac{1}{2}$ (B) $f_1 + f_2 + \frac{1}{3}$

(C) $f_1 + f_2 + \frac{2}{3}$ (D) $f_1 + f_2 + \frac{3}{4}$

14. India and China produce only shirts and phones using only 2 factors of production: either higher skilled labour H or low skilled labour L. Shirts are high skill labour intensive while phones are low skill labour intensive. The production function for each good is identical in both countries. India and China have equal amounts of lower skilled labour, but India has a greater amount of higher skilled labour. Which good will India import?

(A) Shirts

(B) Phones

(C) Both Shirts and Phones

(D) Neither Shirts nor Phones

15. Consider a duopoly with market demand $p = 10 - q$. The cost function of firm 1 is $7q_1$, and that of firm 2 is $2q_2$, where q_i is the quantity produced by firm i, $i = 1, 2$. In equilibrium, firm 2 charges a price of:

(A) 7 (B) 6

(C) 10 (D) 0

16. Virat and Mithali eat rice and drink milk in exactly the same quantities. The price of rice falls. In response, Virat increases the amount of milk but decreases the amount of rice he consumes. Mithali, on the other hand, increases both rice and milk consumption. Both Virat and Mithali spend all their income on eating rice or drinking milk. For Virat's behaviour to be consistent with standard, well-behaved indifference curves, his preferences over rice consumption imply that for him, rice must be a:

(A) Inferior good (B) Giffen good
(C) Luxury good (D) Normal good

17. If the short-run IS-LM equilibrium occurs at a level of income above the natural rate of output, in the long run output will return to the natural rate via

(A) an increase in the price level
(B) a decrease in the interest rate
(C) an increase in the money supply
(D) a downward shift of the consumption function

18. If the short-run aggregate supply curve is steep, the Phillips curve will be:

(A) flat
(B) steep
(C) backward-bending
(D) unrelated to the slope of the short-run aggregate supply curve

19. There are no capital controls between the US and the UK. If the interest rate is higher in the US than in the UK, then we can conclude that

(A) The US dollar is expected to appreciate with respect to the pound (the UK's currency)
(B) The pound is expected to appreciate with respect to the US dollar

(C) The interest rate in the US is expected to increase

(D) The interest rate in the US is expected to decrease

20. Suppose there are two countries, B and C, that have no trade and no financial transactions with any countries except each other. B imports a total of goods worth 10 million bollars from C, where a bollar is a unit of B's currency. B has no exports. Which of the following must be true?

    (A) B has a capital account deficit

    (B) C has a current account deficit

    (C) C is buying assets from B.

    (D) The exchange rate of collars per bollar is bigger than 1, where a collar is a unit of C's currency.

21. Inventory investment can be expected to

    (A) rise when the real interest rate rises, other things being equal

    (B) not depend on the real interest rate, other things being equal

    (C) fall when the real interest rate rises, other things being equal

    (D) depend only on the change in real GDP

22. A cake of size 1 is to be divided among two individuals 1 and 2. Let x_i be the share of the cake going to individual i, $i = 1, 2$, where $0 \leq x_i \leq 1$. The utility functions are $u_1(x_1, x_2) = x_1$, and $u_2(x_1, x_2) = x_2 + |x_1 - x_2|$, where $|a|$ is the absolute value of a. The Pareto optimal cake divisions include:

    (A) $(1, 0)$ (B) $(1/2, 1/2)$

    (C) $(3/4, 1/4)$ (D) None of the above

23. Rohit spends all his money on dosas and filter coffee. He stays in Delhi where each dosa and filter coffee cost the same. He eats 15 dosas and drinks 35 filter coffees in a week. He gets a chance to move to either Chennai or Bangalore. In Chennai, he can just afford to have

40 dosas and 10 filter coffees in a week. Like in Delhi, each dosa and filter coffee cost the same. In Bangalore, he can just afford to have 10 dosas and 20 filter coffees in a week. Here, 2 filter coffees costs the same as 1 dosa. Where will Rohit prefer to stay?

(A) Delhi
(B) Chennai
(C) Bangalore
(D) Indifferent between Delhi and Chennai

24. Consider the IS-LM model with the real interest rate, R, on the vertical axis and output, Y, on the horizontal axis. Now suppose that the central bank chooses R for the economy, based on its own assessment, at $R = \bar{R}$. In this case the LM curve will

(A) not exist
(B) will be horizontal at $R = \bar{R}$
(C) upward sloping like the usual LM curve
(D) None of the other options

25. Consider a supply-demand diagram for the labor market with an upward sloping labor supply curve (L^s) and a downward sloping labor demand curve (L^d). Let the wage be on the vertical axis, and the level of employment (L) be on the horizontal axis. Suppose the wage is rigid above the equilibrium wage at $\bar{w}$, i.e., it fails to adjust to clear the labor market. Then a reduction in labor demand leads to

(A) A larger reduction in employment compared to the case if wages were flexible
(B) A smaller reduction in employment compared to the case if wages were flexible
(C) The same reduction in employment compared to the case if wages were flexible
(D) None of the other options.

ISI 2021 PEB

Group A

1. Let $f : \Re \to \Re$ be a continuously differentiable function which has at least three distinct zeros. (We say x is a zero of f if $f(x) = 0$). Let $g : \Re \to \Re$ be defined as follows: $g(x) = e^{x/2} f(x)$ for all $x \in \Re$.

   (i) Prove that g has at least three distinct zeros.

   (ii) Prove that the function $f + 2f'$ has at least two distinct zeros.

(10+20=30)

2. (i) Consider the following two variable optimization problem:

$$\max_{x,y} \ (x^2 + y^2)$$
$$\text{subject to}$$
$$x + y \leq 1$$
$$x, y \geq 0$$

   Find all solutions of this optimization problem.

   (ii) In a kingdom far, far away, a King is in the habit of inviting 1000 senators to his annual party. As a tradition, each senator brings the King a bottle of wine. One year, the Queen discovers that one of the senators is trying to assassinate the King by giving him a bottle of poisoned wine. Unfortunately, they do not know which senator, nor which bottle of wine is poisoned, and the poison is completely indiscernible. However, the King has 10 prisoners he plans to execute. He decides to use them as taste testers to determine which bottle of wine contains the poison. The poison when taken has no effect on the prisoner until exactly 24 hours later when the infected prisoner suddenly dies. The King needs to determine which bottle of wine is poisoned by tomorrow so that the festivities can continue as planned. Hence he only has time for one round of testing. How can the King administer the wine to the prisoners to ensure that 24 hours from now he is guaranteed to have found the poisoned wine bottle?

(12+18=30)

3. (i) Let A and B be matrices for which the product AB is defined. Show that if the columns of B are linearly dependent, then the columns of AB are linearly dependent.

(ii) Let e_i denote the column vector with three elements, each of which is zero, except for the i-th element, which is 1. Consider a linear transformation $L : \Re^3 \to \Re^3$ with $L(e_1) = e_1$, $L(e_2) = e_1 + e_2$, and $L(e_3) = e_2 + e_3$. Does L map $\Re^3$ onto $\Re^3$? Prove your answer.

(15+15=30)

Group B

4. This question pertains to a situation in which a particular commodity, like rice, is both available at a subsidised rate from a fair price shop (ration shop) and at a higher price from the open market. Suppose a consumer can buy a certain (fixed) quantity of rice at a lower price from the ration shop (that is, there is a ration quota). In addition, he can buy more of rice (assume a uniform quality of rice) from the open market at a higher price. (You may assume that consumers preferences are represented by standard downward sloping, smooth, convex indifference curves.)

   (i) Graphically depict the consumer's equilibrium (assuming he exhausts the ration quota and in addition buys from the open market).

   (ii) Suppose rice is a normal good. What will happen to the quantity of rice purchased from the open market (over and above the ration quota) in equilibrium if there is a cut in the ration quota? Briefly explain.

   (iii) Suppose rice is a normal good. What will happen to the quantity purchased in the open market (over and above the ration quota) if the subsidised price (price at which the ration quota rice could be bought) is increased (but is still lower than the open market price)? Will your conclusion change if rice is an inferior good? Briefly explain.

(10+10+10=30)

5. There are plenty of fish in the Dull Lake. Boats can be hired by fishermen to catch fish and sell it on the fish market. The revenue earned each month from a total of x boats is given by the following expression: Rupees $10,000\{4x - \frac{1}{2}x^2\}$. Each boat costs Rupees 20,000 each month.

   (i) Derive the marginal and average revenue per boat

   (ii) The Dull municipality is considering giving out permits for each boat that fishes so they can track who is fishing from the lake. If these permits are allocated freely, how many boats will fish every month?

   (iii) If total profit is to be the maximum possible, how many boats should fish every month?

   (iv) Dull municipality decides to charge for the permits instead of giving them out for free. What should the per-boat charge for the permit be if total profits are to be the maximum possible?

(5+10+5+10=30)

6. The Shoddy Theater screens movies every week and is located on a university campus which has only students and faculty as residents. It is the only source of watching movies for both faculty and students, and is large enough to accomodate all faculty and students. Faculty demand for movie tickets is given by $500 - 4P_F = Q_F$, where P_F refers to the price of the ticket paid by faculty and Q_F refers to the number of tickets purchased by faculty. Demand by students is described by $100 - 2P_S = Q_S$, where P_S refers to the price of the ticket paid by students and Q_S refers to the number of tickets purchased by students. The cost to service demand equals 500.

   (i) If the price charged is to be the same for faculty and students, what price would Shoddy Theater set in order to maximize its profits?

   (ii) Now imagine that Shoddy Theater decides to charge different prices for faculty and students. What would these prices be, if Shoddy Theater wants to maximize profits?

(15+15=30)

Group C

7. Consider a Solow type economy, producing a single good, according to the production function:

$$Y(t) = K(t)^{\alpha}\ L(t)^{(1-\alpha)}$$

Where, $0 < \alpha < 1$, and $Y(t), K(t)$ and $L(t)$ are the output of the good, input of capital, and input of labour used in the production of the good, respectively, at time t. Capital and labour are all fully employed.

Labour force grows at the exogenous rate, $\eta > 0$, i.e.,

$$\frac{1}{L(t)}\ \frac{dL(t)}{dt} \equiv \eta > 0.$$

Part of the output is consumed and part saved. Let $0 < s < 1$ be the fraction of output that is saved and invested to build up the capital stock. Also assume that there is no depreciation of capital stock.

Then it follows that:

$$sY(t) = \frac{dK(t)}{dt}$$

Where $\frac{d}{dt}$ is the time derivative.

With this above given description of the economy, one can find out the steady state growth rate of Y, for this economy. Growth rate of output is given by: $\frac{1}{Y(t)} \frac{dY(t)}{dt} \equiv g_Y$.

Assume, the economy begins at date 0, from a per capita capital stock, $k(0) \equiv \frac{K(0)}{L(0)} < k^*$, where k^* denotes the steady state per capita capital stock.

(i) Demonstrate formally, whether the growth rate of output (g_Y), at the beginning date 0, is greater, equal or less than the steady state growth rate of output.

(ii) For the same economy, consider, two alternative beginning date scenarios: with per capita capital stock, given by:
Case 1. $k(0)$; Case 2. $k'(0)$. Where, $k(0) < k'(0) < k^*$ Can you compare the beginning date growth rates of output in the two cases?

(iii) Next, consider two Solow type economies, namely, A and B. They are isolated from each other and are working on their own. Both the economies have absolutely the same description as given before, except for the fact that the fraction of income saved in country A, denoted by s_A is greater than the fraction of income saved in country B, denoted by s_B. Let $k^A(0)$ be the initial date per capita capital stock in country A and $k^B(0)$, the initial date per capita capital stock in country B, which are both less than their respective steady state values. Assume, $K^A(0) < K^B(0)$. Can you figure out whether the initial date growth rate of output in country A is greater than, equal or less than the initial date growth rate of output in country B? In case you find the data provided to you is insufficient to make any comment on this, please point it out.

(20+5+5=30)

8. (i) What is the money multiplier? What determines its size? What is the relation between the monetary base, the money multiplier, and the money supply? Which of these variables can the central bank change to change the money supply? What is the direction of change in each case?

(ii) Why might the cash/deposit ratio and the reserve to asset ratio be decreasing functions of the rate of interest? How does an interest-sensitive money supply affect the LM curve? Illustrate with a diagram, comparing this LM curve with the standard LM. How does this change the effectiveness of counter-cyclical fiscal policy (in a closed economy)? Explain.

(15+15=30)

9. (i) What is the difference between the real and the nominal exchange rate? Give an example to explain this to someone who has not studied economics. Is an increase in the real cost of imports an improvement or a deterioration in the terms of trade?

(ii) A small open economy has a government budget surplus and a trade deficit. Explain whether there is a private sector surplus, deficit or balance. Examine the consequences in the short run for output, the trade balance and the government budget balance of a sudden fall in private consumption in this economy (due to an epidemic in the small country) under (a) fixed exchange rates, (b) flexible exchange rates. Use the Mundell-Fleming model with perfect capital mobility. Explain the adjustment mechanisms.

(10+20=30)

ISI 2022 PEA

1. The number of ways in which the word PANDEMIC can be arranged such that the vowels appear together is

(A) $6 \times (3!)(5!)$ (B) $5 \times (3!)(5!)$
(C) $4 \times (3!)(5!)$ (D) $1 \times (3!)(5!)$

2. Consider the functions $f(x) = x^2 - x - 1$ and $g(x) = x + 1$, both defined for all real values of x. Let $\alpha_1 > 0$ be the positive real root and $\alpha_2 < 0$ be the negative real root of the equation $f(x) = 0$. Let $\beta_1 > 0$ be the positive real root and $\beta_2 < 0$ be the negative real root of the equation $f(g(x)) = 0$. After identifying the exact values of $\alpha_1, \alpha_2, \beta_1$ and β_2, identify which one of the following four statements is *incorrect*.

(A) $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 1$
(B) $\alpha_1 + \beta_2 = \alpha_2 + \beta_1 = 0$
(C) $\alpha_1 + \beta_1 = -(\alpha_2 + \beta_2) = \sqrt{5}$
(D) $\alpha_1 + \alpha_2 = -(\beta_1 + \beta_2) = -1$

3. Let the function $f(x) = 1 - \sqrt{1 - x^2}$ be defined only over all x belonging in $[0, 1]$. Then $f(1 - f(x))$ equals

(A) x (B) $1 - x$
(C) x^2 (D) $1 - x^2$

4. Suppose $f(x)$ is increasing, concave and twice differentiable and $g(x)$ is decreasing, convex and twice differentiable. Then the function $G(x) = g(f(x))$ is

(A) increasing and convex
(B) decreasing and convex
(C) increasing and concave
(D) decreasing and concave

5. Let A and B be two non-singular matrices of the same order and let C be a matrix such that $C = BAB^{-1}$. Then for any scalar λ, the value of $\det(C + \lambda I)$ (where I is the identity matrix) is

(A) $\det A$ (B) $\det B$
(C) $\det(A + \lambda I)$ (D) $\det(B + \lambda I)$

6. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function satisfying $f''(x) > 0$ for all $x \in \mathbb{R}$. Furthermore, assume that $f(1) = 1$ and $f(2) = 2$. Then,

(A) $0 < f'(2) < 1$ (B) $f'(2) > 1$
(C) $f'(2) = 1$ (D) $f'(2) = 0$

7. The value of $\lim_{x\to e} \frac{\log_e x - 1}{x - e}$ is

(A) 0 (B) e
(C) $\frac{1}{e}$ (D) None of these

8. Let $f : [0, \infty) \to \mathbb{R}$ be a function such that $f(0) = 0$ and $f''(x) > 0$ for all $x > 0$. Then the function $g : (0, \infty) \to \mathbb{R}$, defined by $g(x) = \frac{f(x)}{x}$, is

(A) increasing in $(0, \infty)$
(B) decreasing in $(0, \infty)$
(C) increasing in $(0, 1]$ and decreasing in $(1, \infty)$
(D) decreasing in $(0, 1]$ and increasing in $(1, \infty)$

9. Let $f : A \to B$ be a function where $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2\}$. How many onto functions can one generate?

(A) $5^2 - 1$ (B) $5^2 - 2$
(C) $2^5 - 1$ (D) $2^5 - 2$

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \dfrac{x}{1+x^2}$ for all $x \in \mathbb{R}$. Then,

(A) $-1 \leq f(x) \leq 1$ (B) $-1 \leq f(x) \leq 1/2$
(C) $-1/2 \leq f(x) \leq 1$ (D) $-1/2 \leq f(x) \leq 1/2$

11. If a 3×3 matrix A has rank 3 and a 3×4 matrix B has rank 3, then the rank of AB is

(A) 3 (B) 4 (C) 6 (D) 7

12. Let $A = \begin{pmatrix} 2 & 0 & 3 & 1 & -1 \\ 2 & 3 & 1 & 0 & -1 \\ 3 & 1 & 2 & 0 & -1 \\ 1 & 2 & 3 & -1 & 0 \\ 2 & 1 & -1 & 0 & 3 \end{pmatrix}$. Which one is an eigenvalue of A?

(A) 2 (B) 1 (C) 3 (D) 5

13. Let A be a 5×5 non-null singular matrix. Then which of the following statement is true?

(A) $A\mathbf{x} = 0$ has only a trivial solution
(B) $A\mathbf{x} = 0$ has 5 solutions
(C) $A\mathbf{x} = 0$ has no solution
(D) $A\mathbf{x} = 0$ has infinitely many solutions

14. A family has two children. What is the probability that both are boys given that at least one is a boy?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

15. Consider two boxes, one containing one black ball and one white ball, the other containing two white balls and one black ball. A box is selected at random, and a ball is selected at random from the selected box. What is the probability that the ball is black?

(A) $\frac{5}{12}$ (B) $\frac{2}{5}$ (C) $\frac{1}{6}$ (D) $\frac{5}{11}$

16. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = (x^2 + 1)^{2022}$, is

(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

17. Consider an economy with two goods X and Y. Let the utility function be given by $u(x, y) = A\sqrt{xy}$ where $A > 0$, $x \geq 0$ is the amount of good X consumed and $y \geq 0$ is the amount of good Y consumed. Suppose that the budget constraint is given by $P_X x + P_Y y \leq M$ where $M > 0$ is the money income of the consumer and P_X and P_Y are the prices of the goods X and Y, respectively. Let $P_X = P_Y > 1$ and let (x^*, y^*) be the equilibrium quantities of this consumer who maximizes utility subject to the budget constraint. Then,

(A) it must always be that $x^* > y^*$
(B) it must always be that $x^* = y^*$
(C) it must always be that $x^* < y^*$
(D) it must always be that $x^* + y^* = M$

18. Consider the utility function $u(x_1, x_2) = 3x_1 + 2x_2$ of a consumer defined for all $x_1 \geq 0$ and $x_2 \geq 0$. Let the price

of good 1 be $p_1 > 0$ and that of good 2 be $p_2 > 0$. Let $M > 0$ be the money income of the consumer. Consider the optimization problem $\max_{x_1 \geq 0, x_2 \geq 0} 3x_1 + 2x_2$ subject to $2x_1 + 3x_2 \leq M$. The associated Lagrangian function for this maximization problem is $L(x_1, x_2; \lambda) = 3x_1 + 2x_2 + \lambda[M - 2x_1 - 3x_2]$; where λ denotes the non-negative Lagrangian multiplier. Then the equilibrium solution $(x_1^*, x_2^*, \lambda^*)$ to this Lagrangian function maximization problem is

(A) $(x_1^* = \frac{M}{2}, x_2^* = 0, \lambda^* = \frac{2}{3})$

(B) $(x_1^* = \frac{M}{2}, x_2^* = 0, \lambda^* = \frac{3}{2})$

(C) $(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{2}{3})$

(D) $(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{3}{2})$

19. Consider a two good economy where the two goods are X and Y and consider two consumers A and B. In a month when the price of good X was Rs. 2 and that of good Y was Rs. 3, consumer A consumed 3 units of good X and 8 units of good Y and consumer B consumed 6 units of both goods. In the next month, when the price of good X was Rs. 3 and that of good Y was Rs. 2, consumer A consumed 8 units of good X and 3 units of good Y and consumer B consumed 4 units of good X and 9 units of good Y. Given this information which one of the following statements is correct?

(A) Both consumers satisfy the weak axiom of revealed preference

(B) Neither consumer satisfies the weak axiom of revealed preference

(C) Consumer A satisfies the weak axiom of revealed preference but not consumer B

(D) Consumer B satisfies the weak axiom of revealed preference but not consumer A

20. Let the production function be $Y(L, K) = min\{2L, K\}$, where L and K are the amounts of labor and capital, respectively. Consider the cost function $C(L, K) = wL + rK$, where $w > 0$ denotes the price of labor and $r > 0$ denotes the price of capital. Suppose that (L^*, K^*) is the combination of labor and capital at which cost is minimized subject to the constraint $Y(L, K) \geq \bar{Y}$. Then,

(A) $L^* = \bar{Y}$ and $K^* = \bar{Y}/2$

(B) $L^* = \bar{Y}$ and $K^* = \bar{Y}$

(C) $L^* = \bar{Y}/2$ and $K^* = \bar{Y}$

(D) None of the other options is correct

21. Suppose that there are two firms 1 and 2 that produce the same good. Let the inverted demand function be $P(q_1, q_2) = 1 - q_1 - q_2$, where firm 1 produces $q_1 \geq 0$ and firm 2 produces $q_2 \geq 0$. Suppose that the cost function of firm $i \in \{1, 2\}$ is given by $c_i(q_i) = \kappa_i q_i$, where $\kappa_i \in \left(0, \frac{1}{2}\right)$. Note that there is no fixed cost for either firm. Then, the Cournot equilibrium profit of firm 2 is

(A) $\frac{(1-\kappa_1+\kappa_2)^2}{9}$ (B) $\frac{(1-\kappa_2+\kappa_1)^2}{9}$

(C) $\frac{(1-2\kappa_1+\kappa_2)^2}{9}$ (D) $\frac{(1-2\kappa_2+\kappa_1)^2}{9}$

22. A non-transitive preference relation can be represented by a utility function

(A) Always

(B) Only if preferences are complete

(C) Only if preferences are complete and convex

(D) Never

23. Which of the following statements is correct in a two-good world?

(A) Diminishing marginal utility of both goods is sufficient for diminishing marginal rate of substitution

(B) Diminishing marginal utility of both goods is necessary for diminishing marginal rate of substitution

(C) Diminishing marginal utility of at least one good is necessary for diminishing marginal rate of substitution

(D) Diminishing marginal utility of at least one good is neither necessary nor sufficient for diminishing marginal rate of substitution

24. Rahul consumes two goods, X and Y, in amounts x and y, respectively. Rahul's utility function is $U(x, y) = \min\{x, y\}$. Rahul makes Rs 200; the price of X and price of Y are both Rs 2. Rahul's boss is thinking of sending him to another town where the price of X is Rs 2 and the price of Y is Rs 3. The boss offers no raise in pay. Rahul, who understands compensating and equivalent variations perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the town is just as pleasant as the old, having to move is as bad as a cut in pay of Rs A in his current location. He also says that he would not mind moving if, when he moved, he got a raise of Rs B. What are A and B equal to?

(A) $A = 30$, $B = 70$ (B) $A = 40$, $B = 50$

(C) $A = 50$, $B = 75$ (D) $A = 60$, $B = 60$

25. Let $U(x, y) = -[(10-x)^2 + (10-y)^2]$ be a utility function of some consumer. All prices are equal to 1, and income is 40. Then the optimal values of x and y will be

(A) 10, 10 (B) 0, 0
(C) 5, 5 (D) None of these

26. Consider a production function be $F(K, L) = \min\{\frac{K}{a}, \frac{L}{b}\}$; $a, b > 0$ and $a \neq b$. For any given $K = \overline{K} > 0$, the marginal productivity of labor is

(A) 0
(B) $\frac{1}{a}$ if $L < (\frac{a}{b})\overline{K}$ and 0 otherwise
(C) $\frac{1}{b}$ if $L < (\frac{b}{a})\overline{K}$ and 0 if $L > (\frac{b}{a})\overline{K}$
(D) None of the above

27. Let $e_i(p_0)$ be the price elasticity of demand for a good X of consumer i ($i = 2, \cdots, N$) at price p_0, given its demand function. Consumers do not consume identical amounts of X at p_0. Then the price elasticity of demand at price p_0 for the aggregate demand function for X is

(A) $\sum_i (e_i(p_0))^2$ (B) $\sum_i e_i(p_0)$
(C) $\frac{\sum_i e_i(p_0)}{N}$ (D) None of these

28. There are m identical competitive firms in an industry. Every firm has the (total) cost function $C(q) = q^2 + 1$, where q is the level of its output, $q \geq 0$. Industry demand for the product is given by $D(P) = a - bP$, where P is price, and $a, b > 0$. Then the short-run equilibrium output of each firm is

(A) 0 (B) $\frac{a}{m+2b}$
(C) $\frac{a}{\frac{m}{2}+b}$ (D) $\frac{a}{m+\frac{b}{2}}$

29. Suppose the (total) cost function for a monopolist is $C = 3q^2 + 800$, where q is its output. The inverse market demand function is $p = 280 - 4q$. What is the price elasticity of demand at the profit maximizing price?

(A) -4.5 (B) -3.5 (C) -2.5 (D) -1.5

30. Consider the Solow growth model with constant average saving propensity s, rate of depreciation δ, and labor supply growth rate n. There is no technological progress. Then, at steady state, the capital-output ratio is

(A) $\frac{s}{n+\delta}$ (B) $\frac{n}{\delta+n}$
(C) $\frac{\delta}{s+n}$ (D) $\frac{1}{s+n+\delta}$

ISI 2022 PEB

Group A

1. Answer the following questions.

   (a) Find all maxima and minima of the function $f(x, y) = xy$, subject to the constraints $x + 4y = 120$ and $x, y \geq 0$.

   (b) Find the points on the circle $x^2 + y^2 = 50$ which are closest to and farthest from the point $(1,1)$.

   (c) For what values of α are the vectors $(0,1,\alpha)$, $(\alpha, 1,0)$ and $(1, \alpha, 1)$ in $\mathcal{R}^3$ linearly independent?

[10+15+5]

2. Let $f: \mathcal{R} \to \mathcal{R}$ be a continuous function.

   (a) Let Q denote the set of rational numbers. Prove that, if $f(Q) \subseteq \{1,2,3, \ldots\}$, then f is a constant function.

   (b) Calculate the value of $f'(0)$ when f is differentiable and $|f(x)| \leq x^2$ for all $x \in \mathcal{R}$.

[20+10]

3. Suppose a set of $N = \{1,2, \dots, n\}$ political parties participated in an election; $n \geq 2$. Suppose further that there were a total of V voters, each of whom voted for exactly one party. Each party $i \in N$ received a total of V_i votes, so that $V = \sum_{i=1}^{n} V_i$. Given the vector $(V_1, V_2, \dots, V_n)$, whose elements are the total number of votes received by the n different parties, define $P_1(V_1, V_2, \dots, V_n)$ as the probability that two voters drawn at random *with replacement* voted for different parties and define $P_2(V_1, V_2, \dots, V_n)$ as the probability that two voters drawn at random *without replacement* voted for different parties. Answer the following questions.

(a) Derive the ratio $\frac{P_2}{P_1}$ as a function of V alone.

(b) Consider the special case where $V_i = \frac{V}{n}$ for all $i \in N$. For this case, find the probabilities P_1 and P_2.

[25+5]

Group B

4. Consider an agent living for two periods, 1 and 2. The agent maximizes lifetime utility, given by:

$$U(C_1) + \frac{1}{(1+\rho)} U(C_2),$$

where $\rho > 0$ captures the time preference, while C_1 and C_2 are the agent's consumption in period 1 and period 2, respectively. The agent supplies one unit of labor inelastically in period 1, earning a wage w. A portion of this wage is consumed in period 1 and rest is saved (denoted s). In period 2 the agent does not work, but receives interest income on the savings. Principal plus the interest income on savings goes to finance period 2 consumption. Thus, $C_1 + s = w$ and $C_2 = (1+r)s$, where r is the rate of interest. Assume that the per period utility function can be represented by (and only by) any positive linear transformation of the form $U(C) = \frac{C^{1-\theta}-1}{1-\theta}$, where $0 < \theta < 1$.

(a) Demonstrate, deriving your claim, how optimal savings, s, would respond to changes in r .

(b) Now suppose, initially, $r = \rho$. What happens to optimal savings, s, if r and ρ increase by the same amount (so that the condition $r = \rho$ continues to hold)?

[20+10]

5. A profit maximizing monopolist produces a good with the cost function $C(x) = cx, c > 0$, where x is the level of output, $x \geq 0$. It sells its entire output to a single consumer with the following utility function:

$$u(y) = \theta\sqrt{y} - T(y);$$

where y is the amount of the good purchased by the consumer and T is the payment made by the consumer to the monopolist to purchase the output; $0 \leq y \leq x$; $\theta > 0$. Suppose

$$T(y) = py + t;$$

where $p \geq 0, t \geq 0$ if $y > 0$, and $T(0) = 0$. Thus, in order to purchase any positive amount of the good, the consumer may have to pay a lump-sum amount t, or a per unit price p, or both.

(a) Find the profit of the monopolist when it can choose any non-negative combination of t and p.

(b) Find the profit of the monopolist when it can choose any non-negative p, but is forced to set $t = 0$. Calculate how this profit relates to the profit derived in part (a) and explain your result.

(c) Calculate when social surplus is higher, explaining your result.

(d) Calculate when consumer's surplus is higher, explaining your result.

[10+10+5+5]

6. Answer the following questions.

(a) Let the input demand functions of a profit-maximizing competitive firm operating at unit level of output be given by:

$$x_1 = 1 + 3w_1^{-(1/2)}w_2^a \text{ and } x_2 = 1 + bw_1^{(1/2)}w_2^c \text{ ;}$$

where w_1 and w_2 are input prices. Find the values of the parameters a, b and c.

(b) Check whether the following data, summarizing the observed input-output choices of a competitive firm under three different output-input price situations, are consistent with the hypothesis of profit maximization by that firm.

	P	w	q	x
Observation 1	50	20	20	25
Observation 2	45	15	24	36
Observation 3	40	20	16	16

Here p and w denote output price and input price, respectively, while q and x denote, respectively, the units of output supplied and input demanded by the firm. Each of the three rows specifies an observation of the output-input price configuration and the output-input choice of the firm under that particular price configuration.

(c) Suppose, for the production function $f(x_1, x_2)$, the cost function of a competitive firm is $c(q; w) = w_1^a w_2^{1-a} q$, where $w = (w_1, w_2)$ is the input price vector and q is the level of output; $\alpha \in (0,1)$. Derive the conditional input demand functions and the production function of the firm.

[10+10+(5 +5)]

Group C

7. Consider a world economy consisting of Home (*H*) and Foreign (*F*). Each of these countries produces a single good that is both consumed domestically and exported. Let Foreign output be the numeraire and let p be the relative price of the *H* produced good. Assume full employment in both countries, so that *H* produces a fixed output Y and *F* produces a fixed output Y^*. Let E be the Home expenditure in terms of its own good and let E^* be the Foreign expenditure measured in terms of the foreign good. We will treat E as a parameter of the model, while E^* is endogenous. Assume that consumers have Cobb-Douglas utility functions with fixed expenditure shares. Let α be the share of expenditure of Home consumers on the Foreign produced good and let α^* be the share of expenditure of Foreign consumers on the Home produced good. Assume further $1-\alpha > \alpha^*$ (i.e., the expenditure share of Home consumers on the Home produced good is greater than the expenditure share of Foreign consumers on the Home produced good). World income equals world expenditure, and goods markets clear.

Now, suppose E falls.

(a) What will happen to p?

(b) What will happen to the trade balance of Home, denominated in units of the Foreign good (i.e., to $p(Y - E)$)?

Prove your claims.

[20+10]

8. Consider the Solow growth model with constant average propensity to save s, labor supply growth rate n, no technological progress and zero rate of depreciation. Let v denote the capital-output ratio.

(a) Prove that, at the steady state, $\frac{s}{v} = n$.

(b) Now suppose that, in some initial situation, $\frac{s}{v} > n$. Explain how market forces will operate to restore, over time, the equality $\frac{s}{v} = n$.

(c) In the process of adjustment in (b), in which direction will the real wage and real rental on capital change? Explain.

[8+16+6]

9. Answer the following questions.

(a) Using a simple Keynesian model of income determination, derive and explain the conditions under which a rise in the marginal propensity to save will reduce aggregate savings in the economy.

(b) Using a model of aggregate demand and aggregate supply, explain how an increase in fuel prices would impact aggregate output, employment and the price level.

[15+15]

ISI 2023 PEA

1. A consumer's budgetary allocation for two commodities x and y is given by m. Her demand for commodity x is given by: $x(p_x, p_y, m) = \dfrac{2m}{5p_x}$. Suppose that her budget allocation (m) and the price of commodity y (p_y) remains the same at (m = Rs. 1000, p_y = Rs. 20) while the price of commodity x (p_x) falls from Rs. 5 to Rs. 4. The *substitution effect* of this price change is given by

   (a) an increase in demand for x from 80 to 100

   (b) an increase in demand for x from 90 to 100

   (c) an increase in demand for x from 80 to 92

   (d) an increase in demand for x from 80 to 90

2. You are given the following partial information about the purchases of a consumer who consumes only two goods: Good 1 and Good 2.

	Year 1			Year 2	
	Quantity	Price		Quantity	Price
Good 1	100	100	Good 1	120	100
Good 2	100	100	Good 2	??	80

   Suppose that the amount of Good 2 consumed in year 2 is denoted by x. Think about the range of x over which you would conclude that the consumer's consumption bundle in year 1 is *revealed preferred* to that in year 2. Also think about the range of x over which you would conclude that the consumer's consumption bundle in year 2 is *revealed preferred* to that in year 1. Which of the following ranges of x ensures that the consumer's behaviour is inconsistent (that is, it contradicts the *weak axiom of revealed preference*)?

   (a) $x \leq 75$

(b) $x \geq 70$

(c) $70 < x < 75$

(d) $75 < x < 80$

3. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are 23 firms, each with cost function, $c_i(q_i) = \frac{q_i^2}{2}$, $i \in 1, 2, \ldots, 23$. The Cournot-Nash equilibrium

   (a) involves each firm producing 3 units

   (b) involves each firm producing 4 units

   (c) involves each firm producing 5 units

   (d) is not well defined

4. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are 10 firms, each with cost function, $c_i(q_i) = q_i$, $i \in 1, 2, \ldots, 10$. The firms compete in quantities. The total deadweight loss is

   (a) $\frac{9^2}{2}$

   (b) $\frac{99^2}{2}$

   (c) $\frac{10^2}{2}$

   (d) $\frac{100^2}{2}$

5. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There is a large number of firms with identical cost functions

$$c_i(q_i) = \begin{cases} 10 + 2q_i, \text{ if } q_i > 0 \\ 0, \text{ otherwise.} \end{cases}$$

   (a) The competitive equilibrium price is 2

   (b) The competitive equilibrium price is 10

(c) The competitive equilibrium price is 2.1

(d) The competitive equilibrium price is not well defined

6. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are two firms, firm 1 and firm 2, with identical cost functions

$$c_i(q_i) = \begin{cases} 0, \text{ if } q_i \leq 10 \\ \infty, \text{ otherwise,} \end{cases}$$

for $i = 1, 2$. The firms simultaneously announce their prices, p_1 and p_2. The demand coming to firm i is:

$$D_i(p_1, p_2) = \begin{cases} 100 - p_i, & \text{if } p_i < p_j \\ \frac{100-p_i}{2}, & \text{if } p_i = p_j \\ 0, & \text{otherwise.} \end{cases}$$

The Bertrand-Nash equilibrium is

(a) $(p_1 = 0, p_2 = 0)$

(b) $(p_1 = 80, p_2 = 80)$

(c) $(p_1 = 20, p_2 = 20)$

(d) $(p_1 = 90, p_2 = 90)$

7. 500 consumers (of health services) are distributed uniformly over the interval $[0, 1]$. The government can set up two hospitals anywhere in the interval. The hospitals provide health services free of cost, but the consumers have to incur the expenses of travelling to the hospital. The travel cost of a consumer who travels a distance d is d. The fixed cost of setting up a hospital is 300, and the marginal cost of servicing an individual is 2. The worth of the health services to an individual is 4. The government can, of course, decide to set up no hospital. The optimal hospital location decision of a welfare maximizing government is:

(a) set up no hospital

(b) set up two hospitals – both at 1/2

(c) set up two hospitals – one at 1/4, the other at 3/4

(d) set up two hospitals – one at 1/3, the other at 2/3

8. There is a unit mass of consumers who buy either one unit of a product or nothing. Consumer valuation, θ, is distributed according to the distribution function $F(\theta)$ defined over $\left[\underline{\theta}, \overline{\theta}\right]$, that is, for any $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, the proportion of consumers with valuation less than or equal to θ is given by $F(\theta)$. Suppose that the inverse demand function for the product is $p(q)$, where p is market price and q is aggregate demand. Then the slope of the inverse demand function is

(a) $p'(q) = -\frac{1}{F'(p(q))}$

(b) $p'(q) = -F'(p(q))$

(c) $p'(q) = -\frac{1}{F'(q)}$

(d) $p'(q) = -F'(q)$

- **Questions 9 and 10 share the following common information.**

Consider an economy where output (income) is demand determined. In this economy λ proportion $(0 < \lambda < 1)$ of the total income is distributed to the workers, and $(1 - \lambda)$ proportion to the capitalists. The capitalists save s_c fraction $(0 < s_c < 1)$ of their income and consume the rest; the workers save s_w fraction $(0 < s_w < 1)$ of their income and consume the rest; also $s_w > s_c$. The aggregate demand consists of total consumption demand and total investment demand. Investment demand is autonomously given at $\overline{I}$ units.

9. Suppose savings propensities of both the workers and capitalists increase. Then, in the new equilibrium,

   (a) aggregate savings increases and income decreases

   (b) aggregate savings decreases and income increases

   (c) aggregate savings remains unchanged and income decreases

   (d) aggregate savings increases and income remains unchanged

10. Suppose savings propensities remain the same but the share of total income distributed to the workers increases. Then, in the new equilibrium,

   (a) aggregate savings increases and income decreases

   (b) aggregate savings decreases and income increases

   (c) aggregate savings remains unchanged and income decreases

   (d) aggregate savings increases and income remains unchanged

- **Questions 11, 12 and 13 are related and share a common information set.** The complete set of information is revealed gradually as you move from one question to the next. Attempt them sequentially starting from question 11.

11. Consider an economy where the aggregate output in the short run is given by $Y = \left(\overline{K}\right)^{\alpha} L^{1-\alpha}$, $0 < \alpha < 1$, where L is the aggregate labour employment and $\overline{K}$ is the aggregate capital stock (which is fixed in the short run). Let P and W denote the aggregate price level and the nominal wage rate, respectively. The producers in the economy maximize profit in a perfectly competitive market.

    In this economy the demand for labour as a function of real wage rate $\left(\frac{W}{P}\right)$ is given by

(a) $L^d = Y^{\frac{1}{1-\alpha}} \left(\overline{K}\right)^{\frac{\alpha}{1-\alpha}}$

(b) $L^d = \overline{K}\,(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{W}{P}\right)^{-\frac{1}{\alpha}}$

(c) $L^d = \overline{K}\,(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{W}{P}\right)^{\frac{1}{\alpha}}$

(d) $L^d = \overline{K}\,(1-\alpha)^{-\frac{1}{\alpha}} \left(\frac{W}{P}\right)^{\frac{1}{\alpha}}$

12. The above economy is characterized by a representative household which takes the aggregate price level and the nominal wage rate as given and decides on its consumption and labour supply by maximizing its utility subject to its budget constraint. The household has a total endowment of $\overline{L}$ units of labour time, of which it supplies L^s units to the market and enjoys the rest as leisure. Its utility depends on its consumption (C) and leisure ($\overline{L} - L^s$) in the following way: $u = C^{\beta} + \left(\overline{L} - L^s\right)^{\beta}$, $0 < \beta < 1$. The only source of income of the household is the wage income and it spends its entire wage earning in buying consumption goods at the price P.

    In this economy the supply of labour as a function of real wage rate $\left(\frac{W}{P}\right)$ is given by

    (a) $L^s = \frac{\overline{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

    (b) $L^s = \frac{\overline{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}$

    (c) $L^s = \overline{L}\left[1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}\right]$

    (d) $L^s = \frac{\overline{L}}{1-\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

13. Given the labour demand and labour supply functions as derived above, the aggregate supply curve (output (Y) supplied as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is

    (a) upward sloping

(b) downward sloping

(c) horizontal

(d) vertical

14. Consider an economy with aggregate income Y and aggregate price level P. The goods market clearing condition is given by the savings-investment equality: $S(Y,r) = I(r)$, where r is real interest rate and $0 < S_Y < 1$, $S_r > 0$, $I_r < 0$. The money market clearing condition is given by the equality of real money supply $(\frac{M}{P})$ and demand for real balances (L): $\frac{M}{P} = L(Y,r)$, where M is the supply of money and $L_Y > 0$, $L_r < 0$. [For any function $f(x,y)$, f_x denotes the partial derivative of f with respect to x.]

The slope of the aggregate demand curve (aggregate output (Y) demanded as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is

(a) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{M}{P^2} S_Y}$

(b) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{\frac{M}{P^2}(S_r - I_r)}$

(c) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{1}{P} S_Y}$

(d) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{\frac{1}{P}(S_r - I_r)}$

15. To test the prediction of the Solow growth model, you run the following linear regression for all the countries in the world:

$$g_i = \alpha + \beta_0 \log y_{i,0} + \beta_1 \log n_i + \beta_2 \log s_i + \gamma X_i + \varepsilon_i,$$

where g_i is the growth rate in per capita real GDP of country i over a certain period, $y_{i,0}$ is per capita real GDP of country i at the beginning of the period under consideration, n_i is population growth rate of country i, s_i is savings rate of country i, X_i stands for a set of other control variables and ε_i is the error term.

The Solow growth model predicts that the expected sign of the regression coefficient β_0 is

(a) positive

(b) negative

(c) zero

(d) inconclusive

16. Consider the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The rank of A is

(a) 0

(b) 1

(c) 2

(d) 3

17. Bowl A contains two red coins; Bowl B contains two white coins; and Bowl C contains a white and a red coin. A bowl is selected uniformly at random and a coin is chosen from it uniformly at random. If the chosen coin is white, what is the probability that the other coin in the bowl is red?

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

18. A girl chooses a number uniformly at random from $\{1, 2, 3, 4, 5, 6\}$. If she chooses n, then she chooses another number uniformly at random from $\{1, \ldots, n\}$. What is the probability that the second number is 5?

(a) $\frac{11}{180}$

(b) $\frac{2}{45}$

(c) $\frac{1}{3}$

(d) $\frac{1}{18}$

19. The cumulative distribution function F of a standard normal distribution satisfies:

$$F(1.4) = 0.92, \quad F(0.14) = 0.555, \quad F(-0.2) = 0.42, \quad F(-1.6) = 0.055$$

A manufacturer does not know the mean and standard deviation of the diameters of ball bearings it produces. However, he knows that the diameters follow a normal distribution with mean μ and standard deviation σ. It rejects 8% of bearings as too small if the diameter is less than 1.8 cm and 5.5% bearings as too large if the diameter is greater than 2.4 cm.

Which of the following is correct?

(a) $\mu = 2$

(b) $\mu = 2.33$

(c) $\mu = 2.08$

(d) $\mu = 2.4$

20. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'(x)$ is strictly increasing in x ($f'(x)$ indicates the derivative of $f(x)$ with respect to x). Suppose $f(\frac{1}{2}) = \frac{1}{2}$ and $f(1) = 1$. Then which of the following is true ?

(a) $f'(\frac{1}{2}) < 1 < f'(1)$

(b) $f'(\frac{1}{2}) < f'(1) < 1$

(c) $1 < f'(\frac{1}{2}) < f'(1)$

(d) None of the above

21. For any non-negative real number x, define $f(x)$ to be the largest integer not greater than x. For instance, $f(1.2) = 1$. Evaluate the following integral

$$\int_0^{\sqrt{5}} f(x^2)dx$$

(a) 5

(b) $4\sqrt{5} - \sqrt{3} - \sqrt{2} - 3$

(c) $4\sqrt{5}$

(d) $4(\sqrt{5} - 2)$

22. The constant term (i.e., the term not involving x) in the expansion of $\left(x + \frac{1}{x^2}\right)^{19}$ is

(a) 1

(b) 19

(c) 171

(d) none of the above

23. Arjun and Gukesh each toss three different fair coins (each coin either lands heads or tails with equal probability and with each outcome independent of each other). Arjun wins if strictly more of his coins lands on heads than Gukesh, and we call the probability of this event p_1. Which of the following is correct?

(a) $p_1 = \frac{1}{3}$

(b) $p_1 = \frac{11}{32}$

(c) $p_1 = \frac{3}{8}$

(d) $p_1 = \frac{13}{32}$

24. How many real solutions are there to the equation $x|x| + 1 = 3|x|$?

(a) 0

(b) 1

(c) 2

(d) 3

25. We are given n positive integers $k_1, \ldots, k_n$ (need not be distinct) such that

$$k_1 + \ldots + k_n = 5n - 4$$
$$\frac{1}{k_1} + \ldots + \frac{1}{k_n} = 1$$

What is the maximum value of n?

(a) 3

(b) 4

(c) 5

(d) 6

26. A monkey starts at the origin $(0, 0)$ on $\mathbb{R}^2$. The monkey covers a distance of 5 units in any direction in one jump. If the monkey can only go to integer coordinates on $\mathbb{R}^2$, then the number of possible locations after its first jump is equal to

(a) 2

(b) 4

(c) 8

(d) 12

27. There is a strip made up of $(n+2)$ squares, where n is a positive integer. The two end squares are coloured black and other n squares are coloured white. A girl jumps to one of the n white squares uniformly at random and chooses one of its two adjacent squares uniformly at random. What is the probability that the chosen square is white?

(a) $1 - \frac{1}{n+2}$

(b) $1 - \frac{1}{n-1}$

(c) $1 - \frac{1}{n}$

(d) $\frac{1}{2} - \frac{1}{n+1}$

28. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max(|x|, x^2), \qquad \forall\, x.$$

Which of the following is true?

(a) f is not continuous

(b) f is continuous but not differentiable

(c) f is decreasing

(d) f is increasing

29. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max(|x|, x^2), \qquad \forall\, x.$$

Define

$$D := \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R}, y \geq f(x)\}.$$

Which of the following is true for D?

(a) D is not convex

(b) $\mathbb{R}^2 \setminus D$ is convex

(c) $\mathbb{R}^2_+ \setminus D$ is convex

(d) None of the above

30. Suppose $f : [-1, 1] \to \mathbb{R}$ is a function such that

$$f(x) = \frac{2 - x^2}{2} f\left(\frac{x^2}{2 - x^2}\right), \qquad \forall\, x \in [-1, 1].$$

Then, $f(-1)$ is equal to

(a) -1

(b) 1

(c) 0

(d) $\frac{1}{2}$

ISI 2023 PEB

Group A (Microeconomics)

1. [30 marks: $4 + 20 + 6$]

Consider a used car market with 600 buyers each willing to buy exactly one used car, and 500 sellers each having exactly one used car. Out of the 500 used cars, 400 are of good quality (*peaches*) and 100 are of bad quality (*lemons*). The monetary valuation of owning a peach is Rs. 100 for a buyer and Rs. 50 for a seller. On the other hand, the monetary valuation of owning a lemon is Rs. 10 for both a buyer and a seller. A seller knows whether the car she owns is a peach or a lemon, whereas a buyer only knows that there are 400 peaches and 100 lemons. Both the buyers and the sellers know the various valuations.

(a) What outcome maximizes the aggregate surplus of the economy? Provide a clear explanation for your answer.

(b) (i) Derive, with a clear explanation, the supply of used cars as a function of price. Draw this supply curve by plotting number of used cars on x-axis and price on y-axis. [You must label all the important points in the figure clearly.]

(ii) Derive, with a clear explanation, the demand for used cars as a function of price. Draw this demand curve in the same figure as in part (i). [You must label all the important points in the figure clearly.]

(iii) Use the demand and supply functions above to find out *all possible* competitive equilibria in the used car market mentioning clearly which types of car, lemon or peach, are bought and sold in each equilibrium.

(c) Now suppose that buyers also know the identity of all cars, that is, whether any given car is a peach or a lemon. Use a similar demand-supply analysis as above to solve for *all possible* competitive equilibria in the used car market in this scenario.

2. [30 marks: $5 + 3 + 3 + 11 + 8$]

Consider an industry with 2 firms – a *private* firm (indexed by r) and a *public* firm (indexed by u) – producing a homogeneous product and competing in quantities. The firms face an inverse demand function $p = a - bQ$, $a > 0$, $b > 0$, where $Q = q_r + q_u$ denotes aggregate output, and q_r and q_u denote the amounts of output produced by the private and public firms respectively. Each firm i faces the total cost of production cq_i, $i = r, u$, $0 < c < a$.

(a) For any q_r and q_u, derive the expressions for (i) private firm's profit, (ii) public firm's profit, (iii) consumer surplus, and (iv) welfare (sum of consumer surplus and producer surplus).

(b) The *private* firm's objective is to maximize its *own profit*. For a given q_u, set up the private firm's maximization problem and derive its optimal choice of output q_r. [This exercise gives you the *reaction function* of the private firm.]

(c) The *public* firm's objective is to maximize *welfare*. For a given q_r, set up the public firm's maximization problem and derive its optimal choice of output q_u. [This exercise gives you the *reaction function* of the public firm.]

(d) Recall that the two firms compete in quantities.

(i) Define the concept of equilibrium in this context and find out the amounts of output, q_r^* and q_u^*, the two firms produce in equilibrium. Find out the expressions of price, profits of the two firms, consumer surplus and welfare in the equilibrium.

(ii) Illustrate this equilibrium by drawing the two reaction functions you have derived in parts (b) and (c) (plot q_u in x-axis and q_r in y-axis). [You must label the important points in the figure clearly.]

(e) Suppose that the marginal cost of the private firm falls to $c_r < c$ while the marginal cost of the public firm remains the same at c. Draw the new reaction functions and explain clearly how the following outcomes change in the new equilibrium (as compared to the old equilibrium): q_r, q_u, Q, price, profits of the two firms, consumer surplus and welfare. [There is no need to derive the exact expressions; just qualitative answers are enough.]

3. [30 marks: $6 + 4 + 20$]

There is a unit mass of consumers all of whom want to purchase at most 1 unit of a good. Consumer v has a valuation v for this good, where $v \in [0, 1]$. Assume that v is uniformly distributed over interval $[0, 1]$ so that the number of consumers with valuation in between a and b, where $0 \leq a < b \leq 1$, is $b - a$. There is a monopoly firm with total cost of producing q units of the good given by $\frac{q}{3}$. The firm does not know the identity of any consumer and hence must charge a uniform price to all the consumers.

(a) For any price p, derive, with a clear explanation, the demand facing the monopoly firm.

(b) Derive, with a clear explanation, the monopoly price and profit level.

(c) Suppose that the firm can, for a cost, get to know whether a consumer belongs to the interval $[0, \frac{4}{5}]$, or to the interval $(\frac{4}{5}, 1]$. What is the maximum amount the firm is willing to pay for this information? Give a clear explanation for your answer.

Group B (Macroeconomics)

1. [30 marks: $4 + 6 + 10 + 5 + 5$]

   Consider an aggregate demand and aggregate supply model where, in the short run, aggregate capital is fixed at the level $\overline{K}$. The aggregate demand curve, aggregate output (Y) demanded as a function of aggregate price level (P), is given by a standard downward-sloping curve. The aggregate supply curve, aggregate output (Y) supplied as a function of aggregate price level (P), is not standard, and the question leads you to derive the aggregate supply curve.

   The aggregate production function is linear in capital and labour (L): $Y = AL + \overline{K}$, $A > 0$. The labour union is very powerful and dictates the minimum aggregate *nominal* wage rate as $\overline{W}$. Each worker is endowed with one unit of labour which they supply inelastically if the producers offer the nominal wage $W > \overline{W}$. A worker does not supply any labour if $W < \overline{W}$. At $W = \overline{W}$, a worker is indifferent between supplying and not supplying her labour endowment. The number of workers available in the economy is fixed at $\overline{L}$.

(a) Derive, with a clear explanation, the aggregate labour supply (L^S) in this economy as a function of the aggregate nominal wage rate, W.

(b) Note that the marginal product of labour is constant, $A > 0$.

   (i) Derive, with a clear explanation, the aggregate labour demand (L^D) in this economy as a function of the real wage rate, $\frac{W}{P}$.

   (ii) Using your answer to part (i) above, derive the aggregate labour demand (L^D) in this economy as a function of the aggregate nominal wage rate, W.

(c) Choose an arbitrary aggregate price level, P, and draw the aggregate labour supply (L^S) and aggregate labour demand (L^D) curves, as functions of W, by plotting labour (L) on x-axis and nominal wage (W) on y-axis. Think about the labour market equilibrium for the arbitrary aggregate price level P that you have chosen.

Note that the equilibrium employment (L^*) in the economy depends on the arbitrary price level P that you choose. Derive, with a clear explanation, the equilibrium employment (L^*) as a function of aggregate price level P.

(d) Derive, with a clear explanation, aggregate output (Y) supplied as a function of aggregate price level (P). Draw this aggregate supply curve by plotting Y on x-axis and P on y-axis.

(e) Recall that the aggregate demand curve is given by a standard downward-sloping curve. Explain the effectiveness of the standard monetary and fiscal policies in this set up.

2. [30 marks: 14 + 16]

Consider the following version of the Solow growth model. The aggregate output at time t, Y_t, depends on the aggregate capital stock (K_t) and aggregate labour force (L_t) in the following way:

$$Y_t = (K_t)^{\alpha}(L_t)^{1-\alpha},\ 0 < \alpha < 1.$$

There is perfect competition in the factor market so that, in equilibrium, each factor is paid its marginal product and the total output is distributed to all the households in the form of wage earnings and interest earnings. Households save a proportion $0 < s < 1$ of their disposable income in every period. All household savings are invested which augment the capital stock over time. There is no depreciation of capital. Population and therefore the aggregate labour force grows at a constant rate $n > 0$.

(a) The government taxes the *interest earnings* at the rate $0 < \tau < 1$. Wage earnings are not taxed. The government uses the collected taxes to fund *government consumption*; in particular, the tax collection is *not* used for investment at all.

(i) Derive, with clear explanations, the expressions for aggregate wage earning, aggregate interest earning and aggregate savings (S_t) of the economy in terms of Y_t.

(ii) Define $k_t \equiv \frac{K_t}{L_t}$, the capital-labour ratio in period t. Derive, with a clear explanation, the law of motion of capital-labour ratio, that is, the equation with k_{t+1} on the left-hand side and k_t on the right-hand side.

(iii) Derive, with a clear explanation, the steady-state level of capital-labour ratio in this economy, k^*, and examine how k^* changes with changes in the tax rate τ.

(b) As in part (a) above, the government continues taxing interest earnings at the rate τ and wage earnings are not taxed. But consider now that the tax revenue collected is used to fund investment by the government so that the capital stock is further augmented by this public investment.

(i) Derive, with a clear explanation, the expression for aggregate investment in this economy.

(ii) Derive, with a clear explanation, the new law of motion of capital-labour ratio.

(iii) Derive the new steady-state level of capital-labour ratio in this economy, k^{**}, and compare it with k^*. Does the comparison make economic sense?

(iv) How does k^{**} change with changes in τ? Compare with the response of k^* and explain the economic reason behind the differential impact.

3. [30 marks: $9 + 5 + 6 + 4 + 6$]

Consider an individual who lives for two periods. In the first period, she earns an wage income W and takes her consumption-savings decision once income is realized. In the second period, she has no wage income but receives the return along with her principal amount of savings, s. The *gross* rate of interest is $R > 1$. Suppose that there is a government which collects an amount T in the form of lumpsum tax from the wage income in the first period (this can be considered as *mandatory savings* of the individual) and returns the amount T in the form of a lumpsum transfer in the second period. Suppose that the utility derived by the individual who consumes c_1 in the first period and c_2 in the second period is given by $u(c_1) + \beta u(c_2)$, where β is a discount factor with $0 < \beta < 1$ and the utility function u is strictly increasing and strictly concave.

(a) Set up the individual's utility maximization problem by specifying her budget constraint clearly. Derive the first-order condition of this utility maximization problem by showing your procedure clearly. Provide a clear economic interpretation of the first-order condition.

(b) Note that $\beta < 1$ implies that the individual is myopic (short-sighted), she puts less weight on future period. Explain intuitively whether a more myopic individual will save more or less than a less myopic individual. Verify your intuition by determining the sign of $\frac{ds}{d\beta}$.

(c) Note also that the individual's personal savings, s, depends on the government mandated savings T. Explain intuitively whether the government mandated savings increases or decreases personal savings. Verify your intuition by determining the sign of $\frac{ds}{dT}$.

(d) One rupee received in benefits in period 2 would require an individual to save an amount $\frac{1}{R}$ (< 1 since $R > 1$) in period 1. Explain intuitively whether the government mandated savings make the individual cut back her personal savings at a rate higher or lower than $\frac{1}{R}$. Verify your intuition.

(e) In the light of your answer to part (b) and from the expression of $\frac{ds}{dT}$ you expect that the rate of change in personal savings in response to a change in T depends on the discount factor β. Prove that more myopic individuals reduce their personal savings at a higher rate.

Group C (Mathematics)

1. [30 marks: $20 + 10$]

Consider the following feasible region C in $\mathbb{R}^2$ for an optimization program:

$$C := \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1, y \leq x\}.$$

(a) Suppose $a \neq 0, b \neq 0$. Show that an optimal solution to

$$\max_{(x,y)\in C} ax + by$$

is either $(0, 0), (1, 0), (1, 1)$. Describe all possible values of a and b for which each of $\{(0, 0), (1, 0), (1, 1)\}$ is an optimal solution.

(b) Suppose a and b are independently drawn from $[-1, 1]$ using a probability distribution with cumulative distribution function (cdf) F. What is the probability that the unique optimal solution to the above optimization problem is $(1, 0)$?

2. [30 marks: $5 + 10 + 15$]

Suppose A, B, C, a, b, c are real numbers and $A \neq 0$, $a \neq 0$. Suppose for all real values of x, the following holds:

$$|ax^2 + bx + c| \leq |Ax^2 + Bx + C|.$$

Suppose $B^2 - 4AC > 0$.

(a) Argue that $|A| \geq |a|$.

(b) Show that $b^2 - 4ac > 0$.

(c) Show that $B^2 - 4AC \geq b^2 - 4ac$.

3. [30 marks: $5 + 5 + 15 + 5$]

Let $\mathcal{F}$ be a class of functions from $[0, \infty)$ to $[0, \infty)$ with the following properties:

- The functions $f_1(x) = e^x - 1$ and $f_2(x) = \ln(x+1)$ are in $\mathcal{F}$.
- If $f(x)$ and $g(x)$ (not necessarily distinct) are in $\mathcal{F}$, then the functions $f(x) + g(x)$ and $f(g(x))$ are in $\mathcal{F}$.
- If $f(x)$ and $g(x)$ are in $\mathcal{F}$ and $f(x) \geq g(x)$ for all $x \in [0, \infty)$, then $f(x) - g(x)$ is in $\mathcal{F}$.

(a) For any positive integer n, show that $h(x) = nx$ is in $\mathcal{F}$.

(b) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the functions $\ln(f(x)+1)$ and $\ln(g(x)+1)$ are in $\mathcal{F}$.

(c) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the function $f(x)g(x) + f(x) + g(x)$ is in $\mathcal{F}$.

(d) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the function $f(x)g(x)$ is in $\mathcal{F}$.